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Design modelling tasks in digital environments

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This research explores the relationships that future mathematics teachers establish between modelling, mathematical content and technology when designing tasks for teaching mathematics. By means of a qualitative analysis, aspects were revealed in the design of tasks that provide evidence for the transversality of mathematical and extra-mathematical content. In the design process characteristics related to the modelling process were identified. Also observed was how the use of Dynamic Geometry Software (DGS), associated with the construction of static and dynamic simulations, can expand and transform the mathematical work initially planned in a task. The findings of this work show the potential of the design of modelling tasks to the development of knowledge for training future teachers.

Keywords: Modelling, Dynamic Geometry Software, tasks, pre-service teacher

Introduction and conceptual framework

The role of the teacher in selecting, designing or modifying tasks has been discussed in different environments, where two elements that frequently stand out are the intended task and the enacted task. This research explores the work that future mathematics teachers carry out during the process of designing the intended task. In particular, in keeping with the work of Sullivan and collaborators, we study the "interactions among aspects of task design: design elements of tasks, the nature of the mathematics that is the focus of the tasks, and the task design processes" (Sullivan, Knott & Yang, 2015, pp. 84). Tasks for learning mathematics can be classified in different ways, depending on the activities they promote. In this work, a modelling task is defined as one that encourages the modelling activity, where modelling is understood as a transition between reality and mathematics in order to address a problem that occurs in real life.

Various elements of teacher knowledge are brought to bear in the process of designing a task (Hill, Ball & Schilling, 2008). Moreover, when the design considers the use of technology and modeling, the range of knowledge required of the teacher is expanded. We have found results that indicate how participating in the design of modelling tasks in digital environments impacts the Technological Pedagogical Content Knowledge (TPACK) of future mathematics teachers (Guerrero-Ortiz, 2021). Different models have also been explored to describe the skills that a teacher should master to teach modelling, and ways have been suggested to operationalize development of these skills (Borromeo, 2018). The review of the literature reveals that designing tasks for learning is in itself a challenge for teachers and, by involving technology and modelling, the situation becomes even more complex as more knowledge is needed.

Based on the fact that the specific introduction of a type of tool promotes concrete changes in the activities that individuals perform (Jacinto & Carreira, 2017), the types of activities that emerge when modelling in digital environments reflect, to a certain extent, the potentials of the tool that individuals recognize, in our case, future mathematics teachers. By this way, we argue that in the modelling process, the actions of individuals reflect part of the mathematical thinking that is brought to bear when they explore ways of approaching the design of a modelling task. While technology itself does not have an objective, in this work we seek to answer the question: What are the characteristics of the modelling tasks designed by future mathematics teachers with the use of technology?

Methodology

By means of a qualitative analysis, we studied the design process and the modelling tasks for teaching mathematics developed by two groups of future mathematics teachers (G1 and G2). Each group consisted of three participants who had to design, implement and analyze a mathematics teaching task whose content and grade level were chosen by them. Before they designed the tasks, the participants had been introduced to problem solving and modelling in a Dynamic Geometric System (DGS) environment. The design of the task thus reflects in part their knowledge of modelling and their knowledge of teaching mathematics in digital environments. The work of these participants was chosen because it highlights the difficulties encountered by those who design modelling tasks, and it accounts of two modes of exploiting the DGS (Swidan & Faggiano, 2021) in teaching tasks. The information gathering instruments consisted of a document on which the participants reported the modelling process employed during the design of the task, and an electronic file containing the construction in GeoGebra. The written document was analyzed using the content analysis method, while the electronic file was analyzed using the "construction protocol" tool, which allows looking back and redoing every step of the geometrical configuration.

For data analysis, the task design process is first described, consisting of two major stages: the research and the pre-service teachers' modelling process. Each of these stages in turn considers some of the actions shown in table 1. These stages highlight crucial aspects of the modelling process undertaken by the participants that have an impact on the task designed. Subsequently, based on how the participants exploited the technology, ways to expand or improve the task are explored.

Research		**Modelling**
- Choose the context - Explore mathematical and extra-mathematical knowledge -Focus on mathematical content	→	-Simplifying and idealizing - Construct models - Working with models - Obtaining mathematical results - Interpreting and validating

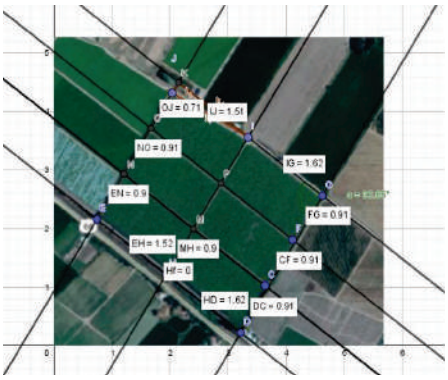
Table 1: Stages of the design process, compiled by authors

Data analysis

This section presents the analysis of the tasks designed by each of the groups. A general description is first shown according to the points listed in Figure 1, and the role of technology in each task is then analyzed, along with the ways in which the potential of DGS can be more fully exploited.

Analysis of design made by Group 1

Based on the school level of the course they were teaching during their professional training (12-13-year-olds), the participants in group G1 decided to address the *teaching of proportions and applications of areas*. Based on this choice of content, they studied the case of a crop farm. They began by exploring in Google Maps (Fig 1) the areas of some tomato, chard and potato farms, which are widely consumed in Chile, from where they obtained relevant data, such as the production yield per hectare, types of soils, climates, and recommended seasons for sowing (extra-mathematical knowledge).

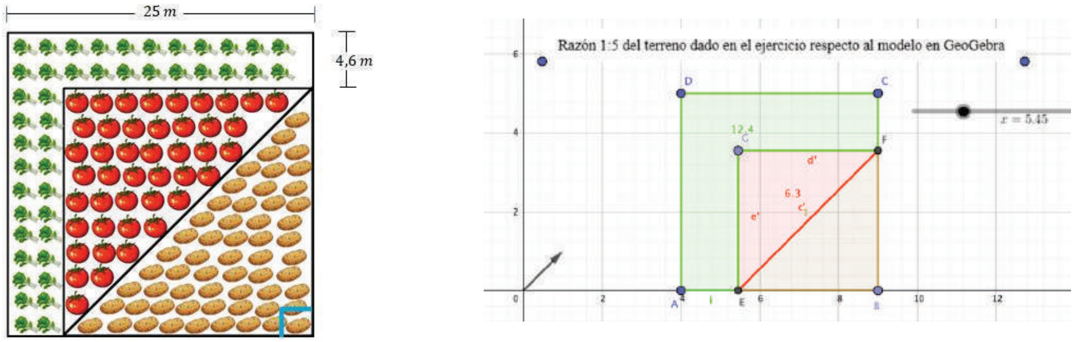


Product	Average yield per hectare	Distance between plants
Chard	16 294	0.4 to 0.5 m
Tomato	25 196	0.5 to 0.7 m
Potato	24 000	0.5 to 0.7 m

Figure 1. Initial exploration of the situation

After exploring the situation, an *idealization* process took place to determine the relevant information to be used in the teaching task, assuming a production yield per square meter. They also considered a square plot measuring 25 m x 25 m, with the condition that each field should have the same area (Fig 2). They also set the values by specifying the production yield per square meter: chard 15 kg/m^2, tomato 8kg/m^2 and potato 13kg/m^2. At this time, the study of the situation lost any semblance with reality, since when calculating the final production, they obtained quantities that differ considerably from the average values. This is explained by the *oversimplification* of the situation, which assumes that each plot of land produces the same amount of product and ignores the density of plants in a given area. This aspect is also related to the omission of the *validation* phase associated with modelling, so the initial approach to this task stands out for the participants' lack of control over their procedures and reflections. This is associated with a metacognitive ability for the development of the modelling process (Czocher, 2018).

In this case, the design of the task depends on the mathematical content for teaching (areas and proportions), an aspect that determined the study context selected and the restrictions in the exploration of a situation that should comprise a modelling task. Moreover, the questions that are finally posed to the students (Fig 2) denote that the activity could well be carried out by omitting the context. In other words, the context is not relevant to achieving the desired objective learning, nor is it a source of reflection. Regarding the DGS *exploitation mode*, since it was initially designed to have the students carry out the construction, it considers the use of tools, such as a polygon, a vector to define the diagonal that divides one of the plots of land, intersections and calculation of distances.



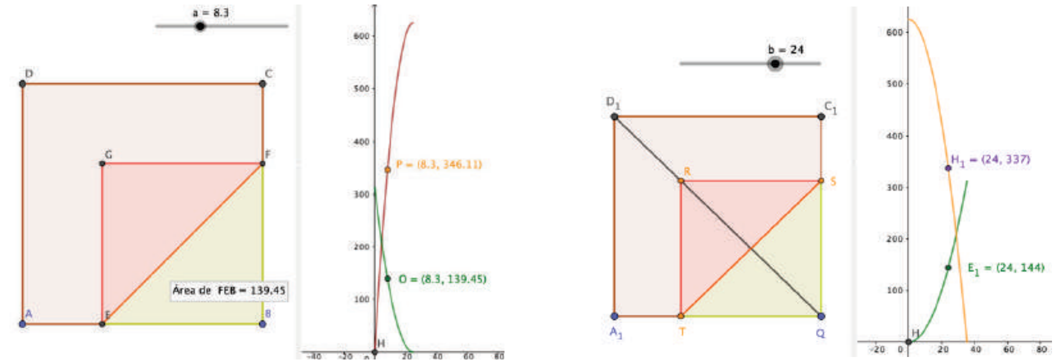
1. Calculate the area of each zone.
2. Calculate the length of the sides of each zone, since you will want to fence them in soon.
3. What is the approximate yield he can expect knowing that [...]
4. If he expands the plot with chard, knowing that the tomato and potato zones have to be the same, how will the proportion change […]

Figure 2. Task designed by a group of future teachers. This figure shows part of task 1 designed by G1 (the introduction has been omitted, which is of the type "Help Mr. Ramón to…").

The mathematical content present in the task, in coordination with the use of technology, can give rise to the exploration of mathematical concepts of another order, for example to introduce the image of simultaneous covariance of two quantities, an element that is essential to understanding the concept of function (Carlson, et al 2002). Based on the idea initially presented in the task, and in view of the fact that the context was irrelevant to the mathematical reflection, we next explore ways, from a purely *mathematical context*, in which the task could be exploited to encourage engagement by the students.

Figure 2 (right side) shows the construction made by the participants, where slider *a* controls the length of side AE, which is congruent to side FC, such that when it is moved, it changes the area of each figure in the geometric configuration. The triangles EFG and EFB are congruent. Graphing the area of one of the triangles and the area of figure AEGFCD yields the right graph in Figure 3a. This can be used to introduce the study of variation, with segment length AE as the independent variable and the area as the dependent variable. The point where the curves intersect represents the case in which all three areas are equal.

Another interesting case that allows us to explore the DGS is the construction where the independent variable is given by the values of segment QR, resulting in the graphical representation shown in Figure 3b. Of note in this case are the different growth rates.



(a) Graph of side AE vs area of triangle FEB and AEGFCD

(b) Graph of side QR vs area of triangle and $C_1D_1A_1TRS$

Figure 3. Exploring different ways to approach the task

These ways of approaching the task, shown in Figures 3 a and b, supported by the dynamism offered by the software, can be used to explore the covariance between the input value and the output value (area) in the same situation. The construction of the dynamic configuration and the use of the slider to control the independent variable are essential to determine the dependency between the variables. This paves the way to introducing the concept of rate of change, domain and range as a preliminary step to studying functions, aspects that had not been initially considered in the design of the task. And improving the context of the task can lead to the study of a crop yield optimization problem. Note that in the previous cases, the sides of the triangle vary similarly, at the same time, giving rise to quadratic behavior. If we vary only one leg of the triangle while keeping the other constant, this results in linear behavior, which introduces a modification to the task. The table 2 below summarizes the main characteristics of this task.

DGS exploitation modes	Predicted mathematical knowledge	Mathematical knowledge emerging from the construction	Technical knowledge
-Construction of a geometric configuration, considering equal areas and different shapes. - Exploration of changes in areas and perimeters, and changes in yield - Static simulation	- Direct and inverse proportions. - Graphic and tabular representation - Lines, perpendicular, parallel, bisector - Triangles, their dimensions - Midpoint of a segment - Congruence - Calculating areas	- Vector - polygon - variable - straight line (vector definition) - point and segment - intersection - symmetry - distance - ratio	- slider to control the characteristics of the configuration. - Point in - Straight line passing through - Vector - Segment - Intersection - Axial symmetry - Distance between points

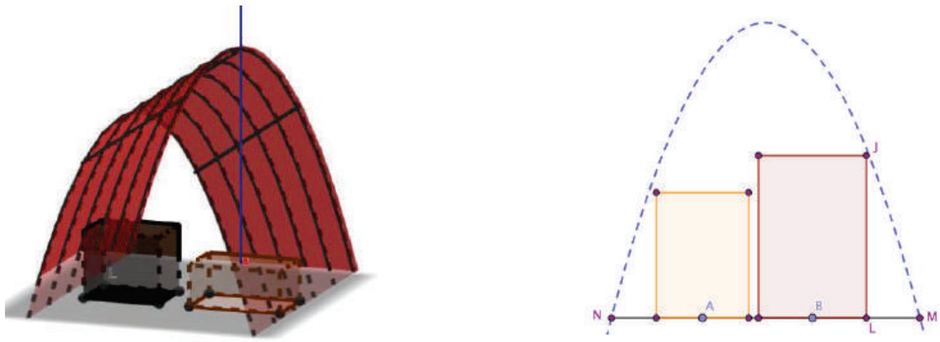
Table 2. Summary of the characteristics

Analysis of design made by Group 2

The design of task 2 is based on a study of the dimensions of vehicles that go through a tunnel. In this case, the participants looked for information online about the dimensions and shapes of tunnels, and about accidents where the vehicle impacts the structure due to exceeding the allowed dimensions. These aspects defined the *choice of context*. Based on the above, the situation was *simplified and idealized* assuming, primarily, that the vehicle moves in a straight line, and the shape of the tunnel was approximated by a parabolic arch with height and base width fixed (Figure 4a). From here, the participants determined the mathematical *learning objective* of the lesson (modelling situations using the quadratic equation, recognizing representations of the parabola and solving problems involving intersections) for high school students (16-17 years old). In this case, as in task 1, there was a certain disconnect with reality due to considering unrealistic data for the dimensions of the vehicles.

The simplification of the situation in this case leads to a study of different representations of the parabola in the context of DGS. On the one hand, by representing the shape of the tunnel with a parabola, the first element that emerges is its construction, which can be carried out in at least three different ways: *a conic given five points*, by means of its *algebraic expression*, and *using the parabola tool given the focus and directrix*. The participants chose the second option. The cars are then represented by means of rectangular prisms.

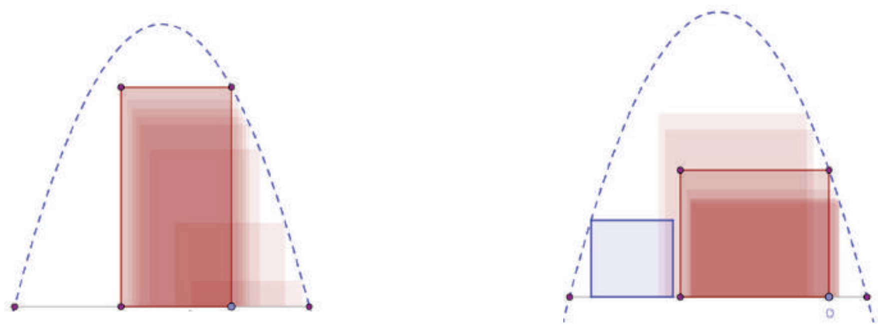
Task 2 asks, given the width, to determine the maximum height that vehicles can have that cross a tunnel shaped like a parabolic arch. This task requires broad mastery of DGS to construct the simulation that represents the situation in the 3D graphic view where the objects are moving (Fig 4a). The analysis is subsequently done in a 2D graphic view. The mathematical activity is then steered to determine the point where the height of the quadrilateral intersects the parabola (Figure 4b). This implies a mathematization process where the shape of the tunnel is represented with a parabola, with its axis on the y-axis, the maximum height of the tunnel (h meters) and its width at the bottom (a meters). These parameters are associated with the vertex of the parabola (0, k) and points (-*a/2*, 0) and (*a/2*, 0) respectively. This information is used to determine its equation $y = -\frac{x^2}{4p} + k$ and graph the parabola and quadrilaterals (Figure 4b).



(a) Representation of the 3D situation **(b) Representation of the 2D situation**

Figure 4. Students' representation of the situation

To explore the solution, if distance LM is fixed in figure 5b, the problem is reduced to determining the equation of the straight line that passes through points J and L, and solving the system of equations formed by the equation of this line and the equation of the parabola. But if this distance is not fixed, another DGS mode of *exploitation emerges* where the quadrilateral has horizontal movement, allowing for exploration to obtain different heights (Figure 5a).



(a) constant base, different heights **(b) allowed dimensions of two quadrilaterals**

Figure 5. Exploring the dimensions of the quadrilateral

Now, since there are two quadrilaterals, exploring the movement allows us to observe how the dimensions of one quadrilateral affect the dimensions of the other such that both can be inscribed in the area between the parabola and the line y = 0. Thus, another form to exploit DGS emerges when modifying the task to allow for exploration of the maximum dimensions that one or both quadrilaterals inscribed in the figure can have (Figure 5b). The characteristics of this task are summarized in Table 3.

DGS exploitation modes	**Predicted mathematical knowledge**	**Mathematical knowledge emerging from the construction**	**Technical knowledge**
-Exploration the intersections using DGS, considering the width and height of the quadrilaterals - Static simulation	- Midpoint - Elements of the parabola: concavity, vertex, endpoints, algebraic expression. - Intersections - Areas	- Line through two points - Segment, distances - Intersection - Multivariate function with domain restrictions - Operations with functions - Parabola and its elements	- Slider to control the characteristics of the configuration. - Point in - Segment - Intersection - Distance between points - 3D graphic

Table 3. Summary of the characteristics of the task designed by group 2.

Discussion and Conclusions

In the above cases, we can differentiate between two elements associated with the design of the tasks. The first is the modelling process undertaken by the future teachers, which involves the following phases: selection of a real-life problem, simplification/idealization, recognition of the mathematical learning objective, and mathematization. When designing a modelling task, it was hoped that the participants would also model it; however, the analysis shows that their process does not span a complete modelling cycle, as defined in some of the commonly recognized modelling cycles (Doerr, Ärlebäck and Misfeldt, 2017). This is explained by the fact that the simplification was done with the goal of designing a task with a specific mathematical learning objective in mind, and not with the goal of teaching modelling.

The second element involves the potential of the software, as exploited by the participants. In both cases, they exhibit a general domain of DGS, their use of which is limited to fulfilling the objective of the tasks. Moreover, representing the situation or part of it in the DGS environment results in a mathematical exercise that, without adequate teacher intervention, could detract from the modelling objective. In the tasks analyzed, the software's role in mediating and enhancing the mathematical activity is evident in the mathematical work phase, a fact that should be taken into account in the pedagogical training of future teachers.

We also observed some tension between the choice of the mathematical learning objective and the choice of an interesting context for the students, in the sense that the choice of the mathematical objective precedes the exploration of a situation to model (task 1), or how the participants sought to determine the mathematical objects in a situation (task 2). This seems to give rise to a context dilemma, in which the solution to the task could either be found without involving a particular context, or the context may be irrelevant or be a source of difficulties in achieving the desired learning (Sullivan, Knott & Yang, 2015).

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