Optimal price and quantity under power demand pattern and non-linear holding cost

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Abstract

In this work we develop a deterministic inventory model for an item whose demand depends on both selling price and time since the last inventory replenishment. More specifically, we assume that the demand rate additively combines the effects of selling price and a time-power function. Moreover, we consider that the holding cost is a power function of the amount of time that a firm holds inventory in stock. The objective is to determine the inventory cycle and the selling price that maximize the total inventory profit per unit time. We present an efficient algorithm to solve this inventory problem. Some numerical examples are provided to illustrate how the algorithm operates.

Keywords: Pricing; Inventory models; Maximum profit; Power demand pattern; Non-linear holding cost

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1 Introduction

Inventory management studies and analyzes the best way to organize the stock of products that a company sells in such a way as to meet customer demand, incurring the minimum possible cost. To do so, it is necessary to implement the most efficient inventory management procedures to guarantee good results. All this requires a set of mathematical models and optimization techniques that allows the best inventory policies to be found.

The main contribution of this paper is to present, discuss and solve a new inventory model that can easily be applied to managing real-life products, for which the consumers' behavior depends on the selling price and the time since the last inventory replenishment. Thus, the inventory model developed in this paper can be useful for products sensitive to price changes, such as: (i) cooked items, fish, fruit or yoghurts, among others, which have a higher demand at the beginning than at the end of the inventory period; (ii) sugar, milk, coffee or oil, among others, which can have a lower demand at the start of the inventory cycle, and (iii) electrical goods, supplies, furniture, kitchen utensils or applicance, among others, which have a quasi-constant demand during the inventory cycle.

Since Harris (1913) published the well-known economic order quantity (EOQ) model, thousands of papers on inventory models have been developed in operations research literature. For recent reviews on mathematical inventory models, we refer the reader to Andriolo et al. (2014), Bazan et al. (2016), Bushuev et al. (2015), Glock et al. (2014) and Shekarian et al. (2017). As in Harris' model, many authors suppose that the demand rate is a known constant. Thus, Yang et al. (2007) developed a collaborative pricing and replenishing policy with finite planning horizon for an inventory system. Also, Gao et al. (2011) studied two bi-level pricing models for pricing problems in a supply chain.

However, in real inventory systems, the demand rate may not be constant and depends on time. Thus, Naddor (1966) introduced the power demand pattern as an adequate function to model the customer demand process. As he noted, it plays a notable role in the inventory management. By using this function, it is assumed that the demand depends on both time and the length of the inventory cycle. There are several works in the literature dealing with the power demand pattern. Goel and Aggarwal (1981) developed an inventory model with power demand pattern for deteriorating items, Datta and Pal (1988) studied an inventory system with power demand pattern and variable rate of deterioration. Lee and Wu (2002) analyzed an inventory system with power demand pattern for deteriorating items, allowing shortages. Dye (2004) presented an inventory model with time-proportional backlogging rate and power demand pattern. Other papers with power demand pattern and partial backlogging are, among others, Singh et al. (2009), Rajeswari and Vanjikkodi (2011), and Mishra and Singh (2013).

A common characteristic of the previous models with a power demand pattern is that they consider a fixed inventory cycle. Sicilia et al. (2012) developed several inventory systems in which the length of the inventory cycle was not fixed. More recently, San-José et al. (2017) studied an inventory system with power demand pattern where the length of the inventory cycle is a decision variable. San-José et al. (2018) developed the optimal policy for an inventory system with full backlogging where demand multiplicatively combines the effects of a price-logit function and a power demand pattern, assuming that the inventory cycle is a decision variable. In this work, we also suppose that the inventory cycle is not fixed and consider that it is a decision variable in the model.

In many inventory systems, it is also assumed that the unit holding cost is a linear function of time in storage. However, this hypothesis may not be realistic for some products. Naddor (1966) analyzed an inventory model in which the holding cost was non-linear with respect to time. Weiss (1982) studied an inventory model with non-linear holding cost and constant demand rate from the perspective of minimizing costs per unit time. Weiss showed that these models with non-linear holding cost can be applicable to any inventory system where the value of the product decreases non-linearly the longer it is held in stock. Later on, Ferguson et al. (2007) revisited the deterministic model analyzed by Weiss (1982) and indicated that it is an approximation of the optimal lot size for perishable goods, such as milk and its derivatives, sold in small- to medium-size grocery stores. Alfares (2007) considered an inventory model for an item with stock-dependent demand rate and storage time-dependent holding cost using two types of discontinuous step functions. Urban (2008) extended the Alfares model from the perspective of maximizing the average profit. Pando et al. (2012) developed an inventory system from the perspective of maximizing profits, but assuming an inventory-level dependent demand rate and power holding cost. Recently, San-José et al. (2015) presented an inventory model with partial backlogging, assuming that the unit holding cost has two significant components: a fixed cost and a variable time-power cost.

One of the main goals of inventory management is to maximize the profit per unit time. Since the profit depends on the selling price, several researchers have considered inventory systems where the demand rate is a function of the unit selling price as a decision variable. Thus, Kunreuther and Richard (1971) investigated the relation between the pricing and inventory decisions when the selling price depends on the quantity sold per unit of time. Smith et al. (2007) analyzed the benefits of joint price and order quantity optimization as compared with a sequential decision process in which the price is determined first, followed by the determination of the order quantity. Kabirian (2012) studied an economic production quantity model in which the demand rate depends on the selling price and the unit purchasing cost is a decreasing function of the lot size. Also, some authors have considered that the demand rate is a function of the marketing parameters and the selling price (see, for example, Bhunia et al., 2015; Mondal et al., 2009 and Shah et al., 2013). However, it is more usual that the demand rate is a function of the selling price and time. In this case, either a multiplicative relation or an additive relation between the effects can be considered. Thus, Chung and Wee (2008) developed an integrated single-retailer/single-manufacturer imperfect production model with partial backordering, warranty-period and stock-level-dependent demand. Yang et al. (2013) analyzed a deteriorating model of a manufacturer purchasing materials and selling products to multi-market with time-varying and price-sensitive demand, considering single and multiple production cycles in a finite time horizon. Panda et al. (2013) developed a deterministic inventory model for perishable items where the demand rate is a function that can be separated into multiplicative effects of price and time. Soni (2013) studied an inventory model with demand influenced by both displayed stock level and selling price for non-instantaneous deteriorating items under delay in payment. His model assumes a demand rate which is additive with respect to both selling price and stock level.Wu et al. (2014) revisited Soni's model and noticed two deficiencies in it. They complemented the shortcomings and developed an optimization procedure to find the optimal replenishment policies. Avinadav et al. (2014) analyzed two models for determining the optimal pricing and the replenishment period for items whose demand function is a separable function of price and time. Wang and Huang (2014) studied a production–inventory problem for a seasonal item, assuming that the demand rate is an additive function of time and price within the selling period. Zhang et al. (2016) studied a decision-making problem for a firm with deteriorating items to jointly determine the sales price, preservation technology, service investments and replenishment policy under an additively separable function of sales price and service level (which obviously depends on time). Recently, Herbon and Khmelnitsky (2017) developed an inventory replenishment model with additive demand rate which generalizes the pseudo-additive model suggested in Avinadav et al. (2014).

In the management literature, a linear price dependence of demand is widely assumed. This is because it is relatively simple to estimate its parameters and the empirical results are easily interpreted. Moreover, it is an advantage that each elasticity of demand depends on the value of the variable (see Oum, 1989 for more details). Alfares and Ghaithan (2016) presented a deterministic inventory model with all-units quantity discounts, where the demand rate is a linearly decreasing function of the selling price and the unit holding-cost is a linearly increasing function of the storage time. Jadidi et al. (2017) studied a joint pricing and inventory decision problem in a single period model with a price-dependent and stochastic demand, where the mean demand varies linearly with the price. Marand et al. (2017) analyzed a service-inventory system in which the arrival rate is modeled as a linear function of the price. More recently, Rubio-Herrero and Baykal-Gürsoy (2018) presented a mean–variance analysis of the single-product, single-period, price-setting newsvendor problem with price-dependent demand in which the expected demand is a linear function of the retailer's price. Other papers on inventory models with linear price-demand have been developed by Bai et al. (2016), Chowdhury et al. (2015), Hong and Lee (2013), Hossen et al. (2016), Maihami and Abadi (2012), Panda et al. (2017) and Zhang et al. (2016). Table 1 summarizes the major characteristics of the previously cited papers that have been published from the year 2000.

In this work, we study a deterministic EOQ model for an item whose demand depends on both selling price and time. More specifically, we suppose that the demand rate additively combines the effects of selling price and a time-power function. Furthermore, we consider that the demand varies linearly with the selling price because this is wholly justified for some products in which demands are lost due to price sensitivity (see Panda et al. (2017)). As we have already commented, this assumption is common in the literature. Moreover, following Weiss (1982), we consider that the holding cost is a power function of the time period in stock. The objective consists in determining the inventory cycle and the selling price to maximize the total inventory profit per unit time. In order to solve the inventory problem, we use a sequential optimization procedure, and based on this, we develop an effective algorithm which finds the optimal selling price and the optimal inventory cycle that determine the maximum profit per unit time. To the best of our knowledge, this is the first work that additively

Table 1. Summary of selected literature from the year 2000

Authors	Objective: maximum profit	Price-dependent demand	Stock-level dependent demand	Power demand pattern	Variable holding cost	Other major characteristics
Alfares (2007)			✓		✓	Without shortage
Alfares and Ghaithan (2016)		✓			✓	Quantity discounts
Avinadav et al. (2014)	✓	✓				Time-dependent demand
Bai et al. (2016)	✓	✓				Supply chain coordination
Bhunia et al. (2015)		✓				Two storage inventory
Chowdhury et al. (2015)	✓	✓				Time-dependent demand
Chung and Wee (2008)		✓	✓			Warranty-period-dependent demand
Dye (2004)				✓		Partial backlogging
Ferguson et al. (2007)					✓	Fixed demand rate
Gao et al. (2011)	✓					Bi-level pricing problem
Herbon and Khmelnitsky (2017)	✓	✓				Time-dependent demand
Hong and Lee (2013)	✓	✓				Time-based policy
Hossen et al. (2016)		✓			✓	Time-dependent demand
Jadidi et al. (2017)	✓	✓				Stochastic demand
Kabirian (2012)	✓	✓				Variable production cost
Lee and Wu (2002)				✓		Partial backlogging
Maihami and Abadi (2012)	✓	✓				Partial backlogging
Marand et al. (2017)	✓	✓				Service-inventory system
Mishra and Singh (2013)				✓		Partial backlogging
Mondal et al. (2009)	✓	✓				Variable production cost
Panda et al. (2013)	✓	✓				Time-dependent demand
Panda et al. (2017)	✓	✓				Production inventory model
Pando et al. (2012)	✓		✓		✓	Without shortage
Rajeswari and Vanjikkodi (2011)				✓		Partial backlogging
Rubio-Herrero and Baykal-Gürsoy (2018)	✓	✓				Newsvendor problem
San-José et al. (2018)	✓			✓		Full backlogging
San-José et al. (2015)					✓	Partial backlogging
San-José et al. (2017)	✓			✓		Partial backlogging
Shah et al. (2013)	✓	✓			✓	Advertisement-dependent demand
Sicilia et al. (2012)				✓		Full backlogging
Singh et al. (2009)				✓		Partial backlogging
Smith et al. (2007)	✓	✓				Without shortage
Soni (2013)	✓	✓	✓			Permissible delay in payment
Urban (2008)	✓		✓		✓	Without shortage
Wang and Huang (2014)	✓	✓				Time-dependent demand
Wu et al. (2014)	✓	✓	✓			Permissible delay in payment
Yang et al. (2007)	✓					Two echelon system
Yang et al. (2013)		✓				Time-dependent demand
Zhang et al. (2016)	✓	✓				Service level-dependent demand
This paper	✓	✓		✓	✓	Without shortage

combines a price-dependent demand and a power-time demand pattern, while also considering a variable inventory cycle and a non-linear holding cost.

The structure of this work is as follows. Section 2 presents the assumptions that characterize the inventory system under study and introduces the notation used throughout the work. Section 3 deals with the mathematical formulation of the proposed model. Then we give the theoretical results and provide the optimal policy in Section 4. Moreover, we show how some models in the inventory literature can be obtained as particular cases from the model studied here. Numerical examples and a sensitivity analysis are shown in Section 5. Finally, the conclusions are described in Section 6.

2 Assumptions and notation

Notation is shown in Table 2.

Table 2. List of notation

Parameters	
K	Ordering cost per replenishment (>0)
p	Unit purchasing cost (>0)
h	Scale parameter of the holding cost (>0)
δ	Elasticity parameter of the holding cost (≥ 1)
α	Scale parameter of the part of the price-dependent demand (>0)
β	Sensitivity parameter of the demand with respect to price (>0)
γ	Scale parameter of the part of the time-dependent demand (>0)
n	Demand pattern index (>0)
Decision variables	
T	Length of the inventory cycle (>0)
s	Unit selling price ($s \geq p$)
Other variables	
q	Lot size per cycle (>0)
Functions	
$H(t)$	Cumulative holding cost per unit held in stock during t units of time
$D(s,t)$	Demand rate at time t for a selling price s, with $0<t<T$
$I(s,t)$	Inventory level at time t for a selling price s, with $0 \leq t < T$
$TP(s,T)$	Total profit per cycle
$B(s,T)$	Profit per unit time

The assumptions used in developing the inventory model are presented below.

1. An inventory system for a single item is considered.
2. The planning horizon is infinite.
3. The replenishment is instantaneous and the item is replenished periodically (each inventory cycle).
4. The purchasing cost p is fixed and known.
5. The selling price s is a constant that must be determined.
6. Shortages are not allowed.
7. The ordering cost K is fixed and regardless of the lot size.
8. The demand rate $D(s,t)$ is a function of the unit selling price and the time that the inventory is held in stock. We consider that $D(s,t) = D_1(s) + D_2(t)$, where $D_1(s)$ is the linear price–demand given by

$$D_1(s) = \alpha - \beta s, \text{ with } \alpha > 0,\ \beta > 0 \text{ and } p \le s \le \alpha/\beta$$

and $D_2(t)$ represents the power–time demand defined as

$$D_2(t) = \left(\frac{\gamma}{n}\right)\left(\frac{t}{T}\right)^{(1-n)/n}, \text{ with } \gamma > 0 \text{ and } n > 0.$$

Thus, α is the scale parameter of the linear price-demand, β is a coefficient of the selling price sensitivity, γ is the scale parameter of the time-dependent demand and n is the index of the power time demand pattern (representing the way in which the units are taken from the inventory in order to satisfy the demand of the customers). Therefore, the demand rate additively combines the effects of the selling price and a time–power function.

9. The cumulative holding cost for a unit held in stock during t units of time is a power function of the time in storage. Thus, we suppose that $H(t) = ht^\delta$, where $h > 0$ is the scale parameter and $\delta \ge 1$ is the elasticity parameter of the holding cost.
10. The lot size per cycle is equal to the total demand throughout the inventory cycle, that is, $q = \int_0^T D(s,t)dt = (\alpha - \beta s + \gamma)T$.

3 Mathematical model

We consider that an order of q units is received at time $t = 0$. During the period $(0,T)$, the inventory level $I(s,t)$ decreases due to demand and drops to zero at $t = T$. Hence, for all $t \in [0,T)$, the inventory level at time

t is given by

$$I(s,t) = q - \int_0^t D(s,u)du = \int_t^T D(s,u)du = (\alpha - \beta s)(T-t) + \gamma T \left[1 - \left(\frac{t}{T}\right)^{1/n}\right].$$

Taking into account the above assumptions, revenue and costs at each inventory cycle are calculated below:

- Revenue: $sq = sI(s,0) = s(\alpha - \beta s + \gamma)T$
- Purchase cost: $pq = p(\alpha - \beta s + \gamma)T$
- Order cost: K
- Holding cost: $\int_0^T H(t)D(s,t)dt = hb(s)T^{1+\delta}$, where for simplicity we define

$$b(s) = \frac{\alpha - \beta s}{1+\delta} + \frac{\gamma}{1+n\delta} > 0. \tag{1}$$

The total profit per cycle $TP(s,T)$ is the difference between the revenue per inventory cycle and the sum of the purchasing cost, the ordering cost and the inventory holding cost per cycle. Then,

$$TP(s,T) = (s-p)(\alpha - \beta s + \gamma)T - \left(K + hb(s)T^{\delta+1}\right). \tag{2}$$

Our objective consists in maximizing the total profit per unit time. So the inventory profit per unit time is given by

$$B(s,T) = \frac{TP(s,T)}{T} = (s-p)(\alpha - \beta s + \gamma) - \left(\frac{K}{T} + hb(s)T^{\delta}\right). \tag{3}$$

Thus, the optimization problem addressed in the paper is

$$\max_{(s,T)\in\Omega} B(s,T), \tag{4}$$

where $\Omega = \{(s,T) : T > 0 \text{ and } p \leq s \leq \alpha/\beta\}$.

4 Solution of the problem

Firstly, we will study the concavity of the function $B(s,T)$. To do this, we calculate the first and second order partial derivatives of $B(s,T)$. So the Hessian matrix of $B(s,T)$, denoted by H_B, is

$$H_B = \begin{pmatrix} \frac{\partial^2 B(s,T)}{\partial s^2} & \frac{\partial^2 B(s,T)}{\partial s \partial T} \\ \frac{\partial^2 B(s,T)}{\partial T \partial s} & \frac{\partial^2 B(s,T)}{\partial T^2} \end{pmatrix} = \begin{pmatrix} -2\beta & \frac{\beta\delta h}{1+\delta}T^{\delta-1} \\ \frac{\beta\delta h}{1+\delta}T^{\delta-1} & -\frac{2K}{T^3} - h\delta(\delta-1)b(s)T^{\delta-2} \end{pmatrix}. \tag{5}$$

Since $\det(H_B) = \frac{4\beta K}{T^3} + 2\beta\delta(\delta-1)hb(s)T^{\delta-2} - \left(\frac{\beta\delta h}{1+\delta}T^{\delta-1}\right)^2$ is not always positive, $B(s,T)$ is neither concave nor convex. For this reason, we will use a sequential optimization procedure to solve the problem (4).

We suppose $s \in [p, \alpha/\beta]$ is fixed and $T > 0$ is variable. Thus, we are considering the univariate function $B_s(T) = B(s,T)$. Taking the derivative of $B_s(T)$, we obtain

$$B_s'(T) = \frac{K}{T^2} - h\delta b(s)T^{\delta-1}$$

and the second derivative is

$$B_s''(T) = -\left(\frac{2K}{T^3} + h\delta(\delta-1)b(s)T^{\delta-2}\right).$$

As $B_s''(T) < 0$ for all $T > 0$, $B_s(T)$ is a concave function. Since $\lim_{T\uparrow 0} B_s(T) = \lim_{T\to\infty} B_s(T) = -\infty$, the maximum of $B_s(T)$ is attained at the point $T^*(s)$, which solves the equation $B_s'(T) = 0$. Thus,

$$T^*(s) = \left(\frac{K}{\delta hb(s)}\right)^{1/(1+\delta)}. \tag{6}$$

Evaluating the function $B(s,T)$ at $T^*(s)$ yields

$$F(s) = B(s, T^*(s)) = (s-p)(\alpha - \beta s + \gamma) - \frac{(1+\delta)K}{\delta T^*(s)}. \tag{7}$$

Also,

$$F(s) = (s-p)(\alpha - \beta s + \gamma) - (1+\delta)hb(s)\left(T^*(s)\right)^{\delta}. \tag{8}$$

Next, we analyze the behavior of the function $F(s)$ to obtain the optimal selling price. It is evident that $F(s)$ is a continuous and differentiable function on the interval $(p, \alpha/\beta)$. Now, taking the derivative of $F(s)$, we obtain

$$F'(s) = \alpha - 2\beta s + \gamma + \beta p + \frac{\beta h}{1+\delta}\left(T^*(s)\right)^{\delta}. \tag{9}$$

From (9) it follows that $F(s)$ is a strictly increasing function when $\gamma \geq \alpha - \beta p$. In this case, it is obvious that the maximum of the function $F(s)$ is attained at the point $s^* = \alpha/\beta$.

Otherwise (that is, if $\gamma < \alpha - \beta p$), we define the point

$$s_o = \tfrac{\alpha+\gamma+\beta p}{2\beta}.$$

It is clear that, in this case, we have $p < s_o < \alpha/\beta$. Note that $b(s_o) = \frac{\alpha-(\gamma+\beta p)}{2(1+\delta)} + \frac{\gamma}{1+n\delta} > 0$ and $F'(s_o) = \frac{\beta h}{1+\delta}\left(\frac{K}{\delta hb(s_o)}\right)^{\delta/(1+\delta)} > 0$.

Since $F'(s) > 0$ for $s \in (p, s_o]$, $F(s)$ is a strictly increasing function on the interval (p, s_o). Next, we study the behavior of this derivative $F'(s)$ when $s \in (s_o, \alpha/\beta)$. Note that $F(s)$ is a twice differentiable function on the interval $(s_o, \alpha/\beta)$. Now, taking the second derivative of $F(s)$, we have

$$F''(s) = -2\beta + \frac{\beta^2 \delta h}{(1+\delta)^3 b(s)} \left(T^*(s)\right)^{\delta}. \tag{10}$$

Consequently, if there exists a solution $\widetilde{s} \in (s_o, \alpha/\beta)$ to the equation $F'(s) = 0$ in the interval $(s_o, \alpha/\beta)$, that solution should satisfy $F''(\widetilde{s}) = -\frac{\beta}{(1+\delta)^2 b(\widetilde{s})} f(\widetilde{s})$, where

$$f(s) = 2(1+\delta)^2 b(s) + \delta(\alpha - 2\beta s + \gamma + \beta p). \tag{11}$$

Hence, $f(s)$ is a strictly linear decreasing function. From this, we deduce that the function $F(s)$ has at most two local extremes in the interval $(s_o, \alpha/\beta)$. Let s_1 be the root of the function $f(s)$. It is easy to check that

$$s_1 = s_o + \tfrac{(1+\delta)^2}{\beta(1+2\delta)} b(s_o). \tag{12}$$

Moreover, it is necessary that $\widetilde{s} < s_1$ so that the function $F(s)$ has a local maximum at the point $\widetilde{s} \in (s_o, \alpha/\beta)$.

The following result provides a criterion for determining the optimal value of the unit selling price.

Theorem 1 *Let $s_o = (\alpha + \gamma + \beta p)/(2\beta)$, $s_1 = s_o + (1+\delta)^2 b(s_o)/(\beta(1+2\delta))$ and $F(s)$, $F'(s)$ and $F''(s)$ be given, respectively, by (7), (9) and (10). The optimal selling price s^* is characterized as follows:*

1. *If $\gamma \geq \alpha - \beta p$, then $s^* = \alpha/\beta$.*
2. *If $\gamma < \alpha - \beta p$ and $F'(\alpha/\beta) < 0$, then s^* is the unique solution to the equation $F'(s) = 0$ in the interval $(s_o, \alpha/\beta)$.*
3. *If $\gamma < \alpha - \beta p$ and $F'(\alpha/\beta) \geq 0$, then the following cases can occur:*
   (a) *If $s_1 \geq \alpha/\beta$, then $s^* = \alpha/\beta$.*
   (b) *If $s_1 < \alpha/\beta$ and $F'(s_1) \geq 0$, then $s^* = \alpha/\beta$.*
   (c) *Otherwise ($s_1 < \alpha/\beta$ and $F'(s_1) < 0$), let $\widetilde{s} = \arg_{s \in (s_o, s_1)}\{F'(s) = 0\}$.*
      i. *If $F(\widetilde{s}) \leq F(\alpha/\beta)$, then $s^* = \alpha/\beta$.*
      ii. *If $F(\widetilde{s}) > F(\alpha/\beta)$, then $s^* = \widetilde{s}$.*

Proof.

1. It is immediate because $F(s)$ is a strictly increasing function on the interval $(p, \alpha/\beta)$.

2. In this case, $F(s)$ has a unique local extreme $\widetilde{s}$ on the interval $(s_o, \alpha/\beta)$. So, $F(s)$ is a strictly increasing function on $(p, \widetilde{s})$ and strictly decreasing on $(\widetilde{s}, \alpha/\beta)$. Therefore, $F(s)$ attains its maximum at $\widetilde{s} = \arg_{s \in (s_o, \alpha/\beta)}\{F'(s) = 0\}$.

3. Note that, in this case, the function $F(s)$ has zero or two local extreme points on the interval $(s_o, \alpha/\beta)$. We can consider the following situations:

   (a) If $s_1 \geq \alpha/\beta$, then the function $F'(s)$ has no roots on the interval considered. Therefore, the function $F(s)$ is strictly increasing in that interval.

   (b) We have divided the proof into two cases:

      i. If $s_1 < \alpha/\beta$ and $F'(s_1) > 0$, then $F'(s)$ has no roots on the interval $(s_o, \alpha/\beta)$. The rest of the proof runs as in the previous case.

      ii. If $s_1 < \alpha/\beta$ and $F'(s_1) = 0$, then $F(s)$ is a non-decreasing function on $(s_o, \alpha/\beta)$. Thus, $F(s)$ attains its maximum at the point $s^* = \alpha/\beta$.

   (c) Finally, if $s_1 < \alpha/\beta$ and $F'(s_1) < 0$, then $F(s)$ has two local extreme points on $(s_o, \alpha/\beta)$: $\widetilde{s}$ and $\widetilde{s}_1$, with $\widetilde{s} < s_1 < \widetilde{s}_1$. Now, the function $F(s)$ is strictly increasing on $(p, \widetilde{s})$, strictly decreasing on $(\widetilde{s}, \widetilde{s}_1)$ and strictly increasing on $(\widetilde{s}_1, \alpha/\beta)$. Therefore, $F(s)$ attains its maximum at point $s^* = \widetilde{s}$ or at point $s^* = \alpha/\beta$. ■

Let us mention some important consequences of the previous results, which allow the optimal inventory cycle T^*, the economic lot size q^* and the maximum profit per unit time B^* to be explicitly determined.

Corollary 1 *If $s^* = \alpha/\beta$, then $T^* = T_o = \sqrt[1+\delta]{K(1+\delta n)/\delta\gamma h}$, $q^* = \gamma T_o$ and $B^* = (\alpha - \beta p)(\gamma/\beta) - (1+\delta)K/(\delta T_o)$.*

Proof. It follows immediately after taking into account (1), (6) and (7). ■

Corollary 2 *If $s^* < \alpha/\beta$, then $T^* = \frac{K\beta}{\delta(1+\delta)b(s^*)(2\beta s^* - \alpha - \gamma - \beta p)}$, $q^* = \frac{K\beta(\alpha - \beta s^* + \gamma)}{\delta(1+\delta)b(s^*)(2\beta s^* - \alpha - \gamma - \beta p)}$ and $B^* = (s^* - p)(\alpha - \beta s^* + \gamma) + \frac{(1+\delta)^2 b(s^*)}{\beta}(\alpha - 2\beta s^* + \gamma + \beta p)$.*

Proof. In this case, we have $F'(s^*) = 0$. From (7), $F'(s)$ can be rewritten as $F'(s) = \alpha - 2\beta s + \gamma + \beta p + \frac{\beta K}{\delta(1+\delta)b(s)T^*(s)}$. The rest of the proof follows immediately. ■

Taking into account the above properties, we can develop an algorithm to solve the inventory problem presented in this paper.

Algorithm

Step 1 If $\gamma \geq \alpha - \beta p$ then go to Step 8.

Otherwise, go to the next step.

Step 2 Calculate $s_o = (\alpha + \gamma + \beta p)/(2\beta)$.

Step 3 If $F'(\alpha/\beta) < 0$, calculate $\widetilde{s} = \arg_{s \in (s_o, \alpha/\beta)}\{F'(s) = 0\}$. Go to Step 9.

Otherwise, go to the next step.

Step 4 Calculate $s_1 = s_o + (1+\delta)^2 b(s_o)/(\beta(1+2\delta))$.

Step 5 If $s_1 \geq \alpha/\beta$ then go to Step 8.

Otherwise, go to the next step.

Step 6 If $F'(s_1) \geq 0$ then go to Step 8.

Otherwise, go to the next step.

Step 7 Calculate $\widetilde{s} = \arg_{s \in (s_o, s_1)}\{F'(s) = 0\}$.

If $F(\widetilde{s}) > F(\alpha/\beta)$ then go to Step 9.

Otherwise, go to the next step.

Step 8 The optimal selling price is $s^* = \alpha/\beta$ and the optimal cycle is $T^* = T_o = \left(\frac{K(1+\delta n)}{\delta \gamma h}\right)^{1/(1+\delta)}$.

The optimal profit is given by $B^* = B_o = (\alpha - \beta p)(\gamma/\beta) - (1+\delta)K/(\delta T_o)$. Stop.

Step 9 The optimal selling price is $s^* = \widetilde{s}$.

From (6), calculate $T^* = T^*(s^*)$ and, from (7), calculate $B^* = F(s^*)$. Stop.

4.1 Particular models

Next, we show how some inventory models developed by other authors can be obtained as particular cases from the model studied here.

(1) If we suppose that $n = 1$ and $\alpha, \beta \to 0$, we obtain the inventory system analyzed by Weiss (1982) and Ferguson et al. (2007).

(2) If we consider $\delta = 1$ and $\alpha, \beta \to 0$, we have the inventory model with power demand pattern without shortages (see Sicilia et al. (2012)).

(3) If we assume that $n = 1$, $\delta = 1$ and $\gamma \to 0$, then we obtain the same model proposed by Kunreuther and Richard (1971) and Smith et al. (2007) when, in their models, a linear demand curve is considered. Moreover, the optimal solution determined by the algorithm developed here coincides with the "simultaneous solution" given by those authors.

(4) If we suppose that $n = 1$, $\delta = 1$ and $\gamma \to 0$, then we obtain the same model proposed by Kabirian (2012) when, in his model, it is assumed that the production cost is constant, demand rate is linear and production rate tends to infinity.

(5) If we assume that $\delta = 1$ and $\beta, \gamma \to 0$, then we have the classical EOQ model proposed by Harris (1913). In this case, the inventory problem 4 is reduced to $\max_{T>0} B_o(T) = (s-p)\alpha - \left(\frac{K}{T} + \frac{h}{2}\alpha T\right)$.

5 Numerical examples

In this section, we present several numerical examples to illustrate how the algorithm operates.

Note that the previous algorithm considers five cases (dependent on the parameters of the system) that must be analyzed to find the optimal selling price. For that, we give five numerical examples that illustrate each of those situations.

Example 1 Let us assume the parameters $\alpha = 120$, $\beta = 1$, $\gamma = 10$, $n = 0.5$, $K = 200$, $p = 40$, $h = 1.05$ and $\delta = 1.5$. In this case, $\gamma < \alpha - \beta p$. By using the algorithm of the previous section, we have $s_o = 85$ and $F'(\alpha/\beta) = -67.3003$. Therefore, the optimal selling price is $s^* = \arg_{s\in(85,120)} \left\{ \frac{28\sqrt[5]{2025}}{3\sqrt[5]{(940-7s)^3}} - 2s + 170 = 0 \right\} =$ 85.6472. From (6), we obtain the optimal inventory cycle $T^* = T^*(s^*) = 2.11779$ and, from (7), the maximum profit per unit time is $B^* = 1867.18$. Finally, the economic order quantity is $q^* = 93.9301$.

Example 2 We now consider an inventory system with the following parameters: $\alpha = 120$, $\beta = 1$, $\gamma = 60$, $n = 25$, $K = 1600$, $p = 35$, $h = 1.5$ and $\delta = 2$. We have $\gamma < \alpha - \beta p$ and calculate the values $s_o = 107.5$, $F'(\alpha/\beta) = 4.50632$, $s_1 = 117.118$ and $F'(s_1) = 0.582939$. Therefore, the optimal inventory policy is $s^* = \alpha/\beta =$ 120, $T^* = 7.68197$, $B^* = 4787.58$ and $q^* = 921.836$.

Example 3 Suppose the same parameters as in Example 2, but change the values of K, γ and p to $K = 1000$, $\gamma = 40$ and $p = 55$, respectively. Again, we have $\gamma < \alpha - \beta p$ and $s_o = 107.5$. Now $F'(\alpha/\beta) = 3.26372$, $s_1 = 116.412$, $F'(s_1) = -2.58107$ and $\widetilde{s} = 113.223$. Since $F(\alpha/\beta) = 2400.49$ and $F(\widetilde{s}) = 2409.99$, we conclude that the optimal unit selling price is $s^* = \widetilde{s}$. Consequently, $T^* = 4.78460$, $q^* = 223.809$ and $B^* = 2409.99$. Note that, in this case, the equation $F'(s) = 0$ has another root at the point $s_2 = 119.249$ in which the function $F(s)$ has a relative minimum ($F(s_2) = 2399.40$).

Example 4 Assume the same parameters as in Example 2, but modify the values of γ and n to $\gamma = 80$ and $n = 2$, respectively. We have $\gamma < \alpha - \beta p$. Next, we calculate $s_o = 117.5$, $F'(\alpha/\beta) = 0.178721$ and $s_1 = 147.8$. Therefore, the optimal inventory policy is $s^* = \alpha/\beta = 120$, $T^* = 3.21830$, $B^* = 6054.26$ and $q^* = 257.464$.

Example 5 Assume the same parameters as in Example 1, but change the value of β to $\beta = 2.8$. Now, we obtain $\gamma > \alpha - \beta p$. By using the algorithm described in the previous section, we see that the optimal inventory policy is $s^* = \alpha/\beta = 42.8571$, $T^* = 3.45712$, $B^* = -67.8478$ and $q^* = 34.5712$. Therefore, the inventory system is non-profitable for any unit selling price.

Figures 1 to 5 depict the profit functions $B(s, T)$ for each of the numerical examples 1 to 5, respectively.

5.1 Sensitivity analysis

Let us consider an inventory system with the assumptions described in Section 2 and the following input data: $\alpha = 120$, $\beta = 1$, $\gamma = 10$, $K = 200$ and $h = 5$.

To analyze the effect of the unit purchasing cost p, the demand pattern index n and the holding cost elasticity δ on the optimal policy, we provide a table containing some calculations that show the behavior of s^*, T^*, q^* and B^* as functions of p, n and δ. More specifically, Table 3 exhibits computational results when $p \in \{26, 36, 40, 44, 60, 70\}$, $n \in \{0.25, 0.5, 1, 2, 4\}$ and $\delta \in \{1, 1.25, 1.5, 3\}$. These results present certain insights into the inventory model studied here. Some issues are the following:

1. With fixed n and δ, the optimal unit selling price s^* and the optimal inventory cycle T^* increase as the unit purchasing cost p increases. However, the economic lot size q^* and the maximum profit per unit time B^* decrease as p increases.

2. With fixed n and p, the optimal unit selling price s^*, the optimal inventory cycle T^* and the economic lot size q^* decrease as the unit holding cost elasticity δ increases.

3. With fixed p and δ, the optimal unit selling price s^*, the optimal inventory cycle T^*, the economic lot size q^* and the maximum profit per unit time B^* increase as the power demand index n increases.

Table 3. Effects of the parameters p, n and δ on the optimal inventory policy

n		$\delta = 1$				$\delta = 1.25$				$\delta = 1.5$				$\delta = 3$			
	p	s^*	T^*	q^*	B^*	s^*	T^*	q^*	B^*	s^*	T^*	q^*	B^*	s^*	T^*	q^*	B^*
0.25	26	79.4872	1.18979	60.0998	2365.60	79.2543	1.10181	55.9121	2375.69	79.0749	1.04932	53.4368	2385.18	78.5431	0.954274	49.1040	2424.26
	36	84.5588	1.24707	56.6682	1885.82	84.3197	1.14758	52.4216	1893.55	84.1342	1.08756	49.8818	1901.22	83.5774	0.973919	45.2119	1934.86
	40	86.5906	1.27245	55.2363	1708.12	86.3488	1.16773	50.9728	1714.89	86.1605	1.10431	48.4124	1721.81	85.5925	0.982377	43.6249	1753.20
	44	88.6243	1.29947	53.7666	1538.54	88.3797	1.18910	49.4907	1544.35	88.1885	1.12202	46.9131	1550.50	87.6087	0.991219	42.0191	1579.60
	60	96.7854	1.42831	47.4406	941.760	96.5272	1.28975	43.1716	943.544	96.3221	1.20461	40.5686	946.536	95.6853	1.03117	35.3842	965.924
	70	101.915	1.53202	43.0267	635.239	101.646	1.36933	38.8261	634.388	101.429	1.26896	36.2550	635.275	100.746	1.06084	31.0337	648.071
0.5	26	79.5242	1.21932	61.5461	2373.62	79.2964	1.13129	57.3604	2384.10	79.1203	1.07869	54.8834	2393.73	78.5914	0.981760	50.4709	2432.03
	36	84.6016	1.28127	58.1674	1894.24	84.3686	1.18144	53.9107	1902.41	84.1870	1.12107	51.3596	1910.26	83.6336	1.00458	46.5789	1943.15
	40	86.6361	1.30888	56.7581	1716.72	86.4008	1.20367	52.4789	1723.95	86.2168	1.13977	49.9028	1731.06	85.6526	1.01451	44.9908	1761.72
	44	88.6730	1.33839	55.3118	1547.34	88.4354	1.22734	51.0141	1553.62	88.2488	1.15963	48.4161	1559.99	87.6730	1.02495	43.3830	1588.37
	60	96.8510	1.48083	49.0878	951.454	96.6025	1.34040	44.7661	953.856	96.4037	1.25369	42.1193	957.148	95.7722	1.07305	36.7280	975.890
	70	101.997	1.59780	44.7428	645.667	101.740	1.43182	40.4628	645.543	101.532	1.32879	37.8285	646.800	100.855	1.10990	32.3484	659.001
1	26	79.5745	1.25956	63.5141	2383.95	79.3509	1.16921	59.2196	2394.28	79.1768	1.11465	56.6501	2403.57	78.6430	1.00948	51.8441	2439.43
	36	84.6604	1.32833	60.2260	1905.11	84.4326	1.22547	55.8413	1913.18	84.2535	1.16257	53.1836	1920.71	83.6949	1.03596	47.9702	1951.11
	40	86.6991	1.35924	58.8564	1727.83	86.4695	1.25063	54.4405	1734.99	86.2882	1.18392	51.7515	1741.79	85.7186	1.04760	46.3891	1769.93
	44	88.7406	1.39246	57.4522	1558.71	88.5092	1.27759	53.0082	1564.94	88.3256	1.20675	50.2904	1571.02	87.7442	1.05993	44.7883	1596.86
	60	96.9446	1.55569	51.4240	964.098	96.7055	1.40887	46.9077	966.567	96.5115	1.31706	44.1063	969.626	95.8736	1.11809	38.1564	985.735
	70	102.117	1.69386	47.2294	659.370	101.873	1.51854	42.7120	659.422	101.671	1.40822	39.8934	660.503	100.988	1.16440	33.7830	670.009
2	26	79.6302	1.30415	65.6898	2394.63	79.4079	1.20849	61.1399	2404.12	79.2331	1.14990	58.3771	2412.60	78.6869	1.03196	52.9530	2445.12
	36	84.7264	1.38111	62.5280	1916.40	84.5004	1.27165	57.8595	1923.65	84.3208	1.20381	54.9889	1930.36	83.7481	1.06176	49.1082	1957.28
	40	86.7702	1.41604	61.2155	1739.39	86.5426	1.30019	56.5027	1745.74	86.3609	1.22807	53.5917	1751.72	85.7764	1.07499	47.5398	1776.33
	44	88.8173	1.45384	59.8729	1570.56	88.5884	1.33097	55.1176	1576.00	88.4045	1.25417	52.1680	1581.25	87.8074	1.08911	45.9522	1603.50
	60	97.0546	1.64365	54.1509	977.419	96.8204	1.48429	49.2484	979.147	96.6269	1.38328	46.1645	981.381	95.9684	1.15716	39.3800	993.613
	70	102.263	1.81059	50.2198	673.955	102.027	1.61752	45.2472	673.329	101.827	1.49431	42.0998	673.595	101.117	1.21346	35.0488	678.996
4	26	79.6794	1.34351	67.6064	2403.45	79.4554	1.24099	62.7256	2411.79	79.2780	1.17768	59.7343	2419.32	78.7176	1.04714	53.6997	2448.82
	36	84.7854	1.42831	64.5803	1925.76	84.5577	1.31036	59.5459	1931.84	84.3753	1.23671	56.4243	1937.58	83.7860	1.07939	49.8827	1961.33
	40	86.8339	1.46714	63.3307	1749.00	86.6049	1.34198	58.2355	1754.16	86.4202	1.26350	55.0629	1759.16	85.8179	1.09381	48.3268	1780.53
	44	88.8868	1.50942	62.0571	1580.44	88.6563	1.37630	56.9012	1584.69	88.4694	1.29250	53.6783	1588.94	87.8531	1.10928	46.7526	1607.88
	60	97.1580	1.72638	56.6979	988.645	96.9229	1.55081	51.2964	989.166	96.7259	1.43885	47.8763	990.354	96.0403	1.18512	40.2462	998.905
	70	102.406	1.92477	53.1121	686.394	102.170	1.70832	47.5425	684.558	101.966	1.56942	43.9969	683.742	101.220	1.24984	35.9700	685.150

4. Note that, in general, the optimal profit B^* is more sensitive to changes in the purchasing cost p than to changes in the parameters δ and n.

In order to evaluate the sensitivity of the optimal selling-policy of the inventory model with respect to the input parameters of the model K, h and γ, Table 4 shows the obtained results when $\alpha = 120$, $\beta = 1.25$, $p = 40$, $n = 2$, $\delta = 1.25$, $K \in \{200, 300, 400, 500, 600\}$, $h \in \{0.5, 0.75, 1, 1.25, 1.5, 1.75\}$ and $\gamma \in \{10, 20, 30, 40\}$. These results allow the following conclusions to be established:

(i) With fixed K and γ, the optimal unit selling price s^* increases as the scale parameter of the holding cost h increases. However, the optimal inventory cycle T^*, the economic lot size q^* and the maximum profit per unit time B^* decrease as h increases.

(ii) With fixed h and γ, the optimal profit per unit time B^* decreases as the ordering cost K increases. However, the optimal selling price s^*, the optimal inventory cycle T^* and the economic lot size q^* increase as K increases.

(iii) With fixed K and h, the optimal inventory cycle T^* decreases as the parameter γ increases. However, the optimal selling price s^*, the economic lot size q^* and the optimal profit per unit time B^* increase as γ increases.

(iv) In general, the optimal selling price s^* is not very sensitive to changes in the parameters K, h and γ. In fact, the changes with respect to the parameters K and h are very small.

6 Conclusions

An inventory model for a single item whose demand depends on both selling price and time is developed. More specifically, we suppose that the demand rate is the sum of a linear function with respect to the unit selling price and of a power-time function. Furthermore, we assume that the holding cost is a power function of the amount of time in stock. The goal is to maximize the total inventory profit per unit time. This objective function can have several local optimum points. To solve the problem, we develop an effective algorithm that analyzes all possible cases that can occur in the inventory system and finds the global maximum. Although, in general, the optimal solutions cannot be expressed in closed form, they can be obtained easily by using some numerical

Table 4. Effects of the parameters K, h and γ on the optimal inventory policy

K		$\gamma = 10$				$\gamma = 20$				$\gamma = 30$				$\gamma = 40$			
	h	s^*	T^*	q^*	B^*	s^*	T^*	q^*	B^*	s^*	T^*	q^*	B^*	s^*	T^*	q^*	B^*
200	0.5	72.5897	3.80099	149.238	1184.85	76.5768	3.73455	165.362	1523.19	80.5647	3.67178	180.997	1901.56	84.5533	3.61235	196.181	2319.96
	0.75	72.7078	3.18005	124.389	1166.17	76.6922	3.12411	137.882	1504.17	80.6776	3.07129	150.963	1882.21	84.6639	3.02131	163.665	2300.30
	1	72.8058	2.80267	109.284	1150.74	76.7881	2.75312	121.178	1488.46	80.7714	2.70634	132.707	1866.24	84.7556	2.66209	143.900	2284.05
	1.25	72.8914	2.54147	98.8271	1137.36	76.8716	2.49632	109.615	1474.84	80.8531	2.45373	120.070	1852.37	84.8356	2.41344	130.218	2269.96
	1.5	72.9681	2.34649	91.0200	1125.41	76.9465	2.30463	100.982	1462.67	80.9263	2.26515	110.635	1840.00	84.9073	2.22783	120.004	2257.38
	1.75	73.0381	2.19353	84.8949	1114.53	77.0150	2.15426	94.2086	1451.60	80.9932	2.11722	103.233	1828.73	84.9727	2.08222	111.990	2245.92
300	0.5	72.7409	4.56231	178.267	1160.95	76.7246	4.48191	197.627	1498.86	80.7093	4.40601	216.394	1876.81	84.6949	4.33419	234.616	2294.81
	0.75	72.8898	3.81888	148.508	1137.61	76.8700	3.75105	164.718	1475.09	80.8515	3.68705	180.428	1852.63	84.8341	3.62652	195.677	2270.23
	1	73.0136	3.36710	130.418	1118.34	76.9910	3.30689	144.714	1455.48	80.9698	3.25011	158.566	1832.68	84.9498	3.19644	172.009	2249.93
	1.25	73.1216	3.05441	117.894	1101.63	77.0965	2.99947	130.865	1438.47	81.0729	2.94769	143.431	1815.37	85.0507	2.89876	155.625	2232.33
	1.5	73.2187	2.82101	108.543	1086.72	77.1912	2.77000	120.526	1423.28	81.1654	2.72194	132.132	1799.91	85.1412	2.67655	143.392	2216.62
	1.75	73.3074	2.63794	101.207	1073.16	77.2778	2.59001	112.414	1409.47	81.2500	2.54487	123.267	1785.86	85.2239	2.50224	133.795	2202.32
400	0.5	72.8715	5.19520	202.149	1140.46	76.8522	5.10301	224.200	1478.00	80.8341	5.01602	245.572	1855.59	84.8170	4.93375	266.318	2273.23
	0.75	73.0472	4.35054	168.327	1113.13	77.0238	4.27260	186.799	1450.17	81.0018	4.19912	204.697	1827.28	84.9812	4.12966	222.067	2244.45
	1	73.1935	3.83727	147.766	1090.59	77.1666	3.76798	164.064	1427.21	81.1414	3.70269	179.852	1803.92	85.1177	3.64101	195.169	2220.69
	1.25	73.3213	3.48206	133.531	1071.04	77.2913	3.41874	148.325	1407.31	81.2632	3.35911	162.651	1783.66	85.2368	3.30281	176.548	2200.09
	1.5	73.4361	3.21694	122.903	1053.61	77.4033	3.15807	136.573	1389.55	81.3726	3.10266	149.809	1765.59	85.3438	3.05036	162.646	2181.71
	1.75	73.5412	3.00900	114.563	1037.75	77.5058	2.95361	127.353	1373.40	81.4727	2.90150	139.734	1749.14	85.4415	2.85235	151.740	2164.98
500	0.5	72.9888	5.74746	222.795	1122.19	76.9668	5.64483	247.196	1459.39	80.9461	5.54803	270.840	1836.66	84.9266	5.45652	293.788	2253.99
	0.75	73.1887	4.81492	185.442	1091.31	77.1620	4.72800	205.893	1427.96	81.1369	4.64609	225.702	1804.67	85.1132	4.56872	244.922	2221.46
	1	73.3554	4.24830	162.734	1065.85	77.3246	4.17090	180.785	1402.03	81.2957	4.09802	198.264	1778.28	85.2686	4.02923	215.218	2194.62
	1.25	73.5011	3.85619	147.012	1043.79	77.4667	3.78537	163.402	1379.55	81.4345	3.71872	179.268	1755.41	85.4042	3.65585	194.655	2171.35
	1.5	73.6320	3.56356	135.272	1024.11	77.5944	3.49763	150.423	1359.51	81.5591	3.43563	165.086	1735.00	85.5261	3.37717	179.302	2150.59
	1.75	73.7520	3.33406	126.061	1006.22	77.7113	3.27196	140.239	1341.27	81.6733	3.21359	153.958	1716.44	85.6375	3.15859	167.257	2131.71
600	0.5	73.0965	6.24321	241.171	1105.51	77.0720	6.13107	267.683	1442.41	81.0489	6.02535	293.368	1819.38	85.0272	5.92546	318.292	2236.42
	0.75	73.3189	5.23216	200.661	1071.41	77.2890	5.13703	222.890	1407.69	81.2609	5.04744	244.417	1784.04	85.2346	4.96286	265.299	2200.48
	1	73.5043	4.61789	176.032	1043.30	77.4699	4.53307	195.659	1379.05	81.4376	4.45325	214.660	1754.90	85.4073	4.37795	233.086	2170.83
	1.25	73.6666	4.19285	158.979	1018.95	77.6281	4.11513	176.806	1354.24	81.5920	4.04206	194.059	1729.64	85.5582	3.97316	210.785	2145.14
	1.5	73.8126	3.87567	146.245	997.231	77.7704	3.80323	162.729	1332.11	81.7309	3.73518	178.677	1707.11	85.6938	3.67105	194.135	2122.22
	1.75	73.9465	3.62694	136.253	977.492	77.9008	3.55863	151.683	1312.00	81.8580	3.49450	166.609	1686.63	85.8180	3.43411	181.072	2101.38

method to solve the non-linear equations, e.g., the bisection method. Several numerical examples are provided to illustrate how the algorithm works to obtain the optimal inventory policy.

The proposed inventory model can be implemented for real-life products that have a demand pattern with the characteristics described in the introduction. Thus, demand for cooked products, fish, fruit and yoghurts, among others, which have (for a fixed price) higher demand at the beginning than at the end of the inventory cycle, can be considered in the model, assuming a demand pattern index greater than one. Also, there are other products where demand, for a fixed price, is lower at the beginning of the inventory cycle. Thus, household goods such as sugar, milk, coffee and oil, among others, have major demand when the amount in the inventory decreases significantly. In this case, the fluctuation of demand can be modeled considering a demand pattern index less than one. Lastly, other products have, for a fixed price, a constant demand during the inventory cycle. For instance, electrical goods, supplies, furniture, kitchen utensils and appliances, etc. This situation can be modeled by using a demand pattern index equal to one. We present an efficient procedure which finds the optimal selling price and the optimal inventory cycle that determine the maximum profit per unit time for any demand pattern index. Consequently, the inventory model studied in the paper gives insights into inventory management and can help managers in decision-making, providing greater efficiency in logistic operations.

Some future research lines related to this paper could be the following: (a) to develop the inventory model allowing shortages; (b) to analyze the inventory system considering deteriorating items; (c) to study the inventory system assuming discounts in purchasing costs; (d) to consider that the selling price depends on the time since the last inventory replenishment and (e) to develop the inventory system under the assumption of stochastic demand.

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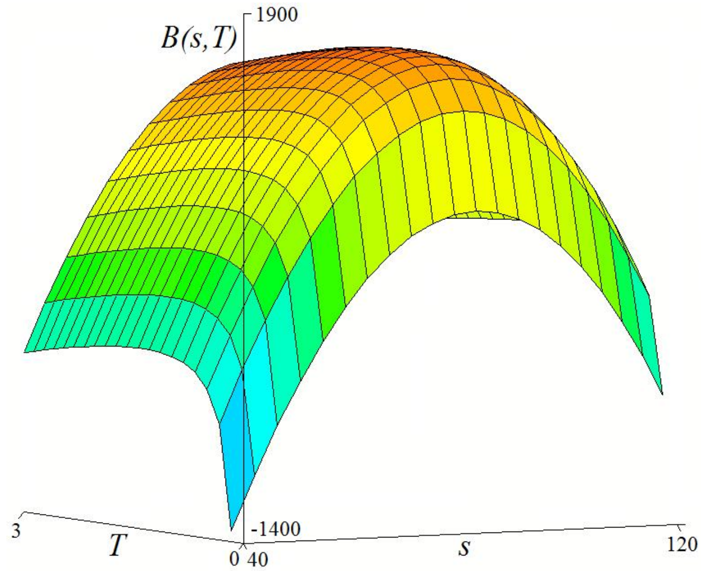

Fig. 1. Graphic of $B(s,T)$ for Example 1

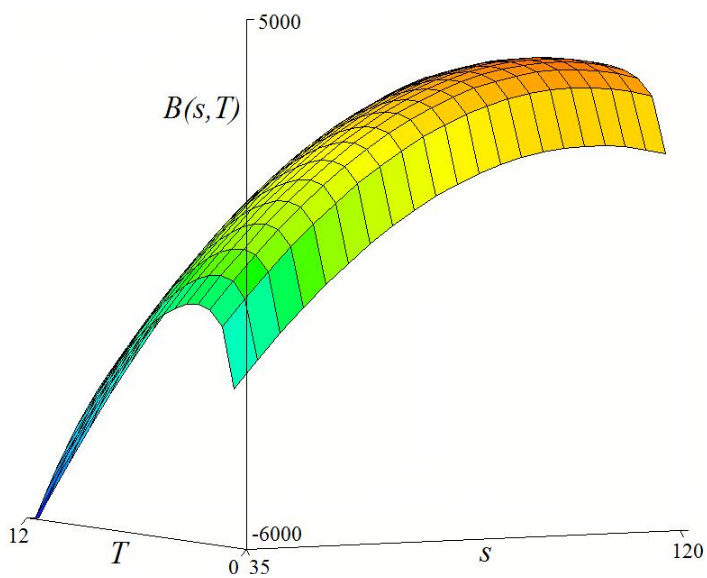

Fig. 2. Graphic of $B(s,T)$ for Example 2

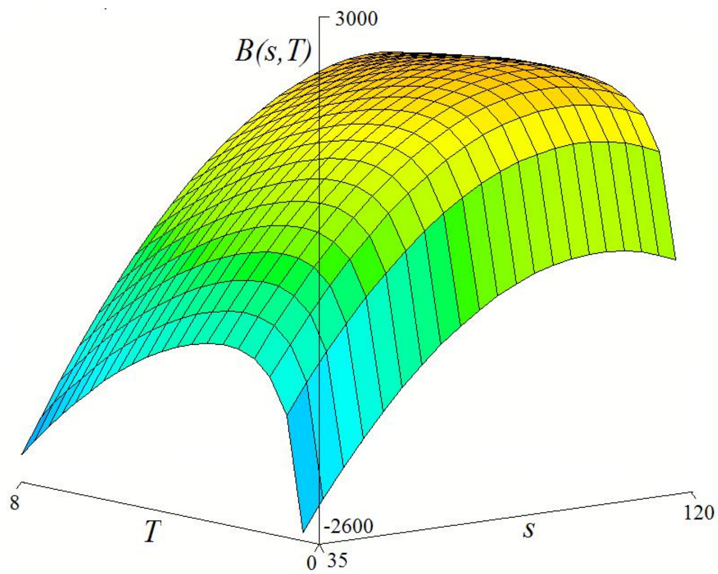

Fig. 3. Graphic of $B(s,T)$ for Example 3

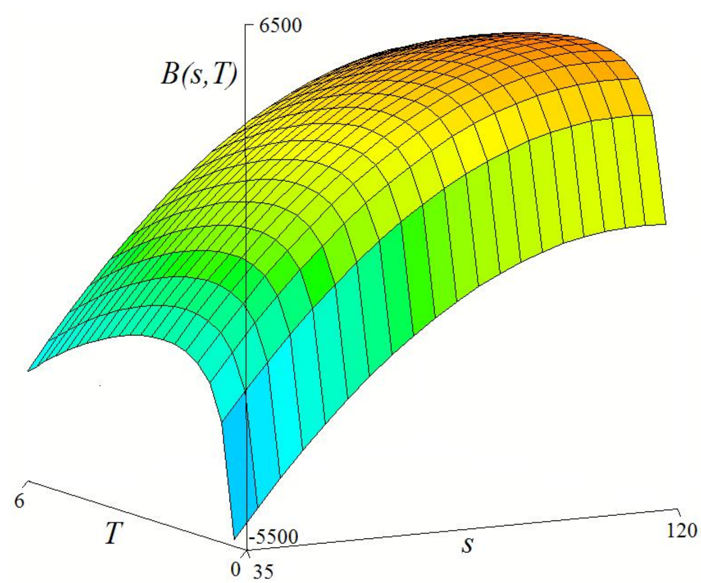

Fig. 4. Graphic of $B(s,T)$ for Example 4

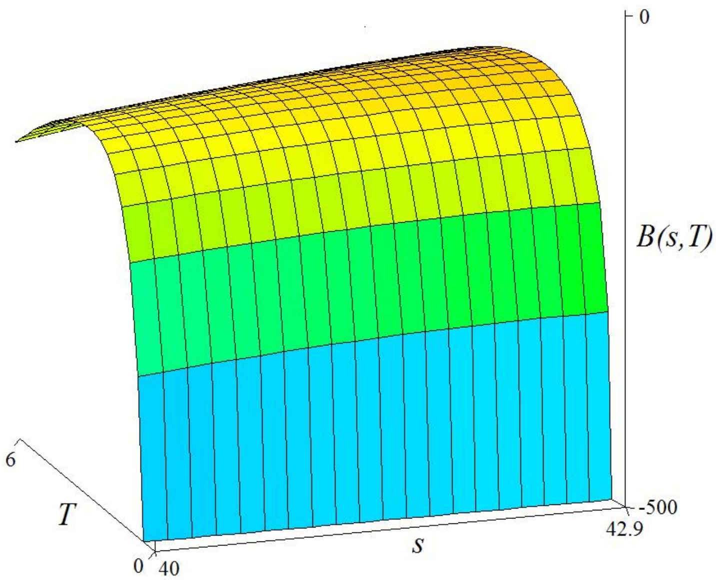

Fig. 5. Graphic of $B(s,T)$ for Example 5