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The influence of problem-posing task situation: Prospective primary teachers working with fractions

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A B S T R A C T

This research presents a study on the problems posed by pre-service primary school teachers by focusing on the problem-posing tasks situation as the research variable. The investigation was carried out with 205 students of a bachelor's degree in Primary Education Teacher in Spain. They were asked to pose problems with fractions based on two given initial situations: numerical and contextualized. For each problem, we analyze its plausibility, the meanings of fractions, the mathematical structure, and the reasonability of the context. Results indicate that mostly posed problems use part-whole or operator meaning of fractions, as well as the additive or multiplicative structure. There are no differences between the plausibility and reasonability of the problems based on the initial situation, although it has shown better results when the given situation is contextualized. In addition, in contextualized situations, teachers show greater ability in formulating problems with a wide variety of structures and meanings of fractions.

1. Introduction

Problem solving is one of the main activities in the development of mathematical knowledge, and consequently one of the priority goals of mathematical education (NCTM, 2000). Niss (2003) describes competence in problem solving and posing as the ability to "identify, create and pose different types of mathematical problems (open, closed; pure or applied) and know how to solve them, whether they were posed by another or by themselves". It thus shows the important link between problem solving and problem posing. Despite this, mathematics education has placed more emphasis on how to solve problems, rather than on how to formulate them (Koichu, 2020).

In recent years, many researchers have turned their attention to problem posing, exploring various aspects of this activity, such as the nature of this task or its role and implementation in math classrooms. This interest is reflected in publications of special issues journals (Cai & Hwang, 2020; Cai & Leikin, 2020; Singer et al., 2013) and books (e.g., Felmer et al., 2016; Singer et al., 2015). Those

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publications conducted so far on this topic point out its importance and the need to conduct further investigation and future lines of research.

Problem posing is an intellectual activity and an efficient way to learn mathematics (Cai et al., 2015). An individual, when formulating a problem, attains complex levels of reflection, achieving reasoning that makes it possible to build mathematical knowledge (Ayllón & Gómez, 2014). Moreover, formulating math problems is a regular activity of teaching professionals (Leavy & Hourigan, 2022b) and it can be an educational tool (Zhang & Cai, 2021). In any case, whether problem posing is considered as an individual intellectual activity or as a professional activity, it is worth determining whether teachers (pre-service or in-service) pose quality mathematical problems, as this would have an impact on their work with students.

In this paper, we analyze different characteristics of fraction problems posed by pre-service primary education teachers like plausibility, reasonability, meanings or the mathematical structure of the problem. We also want to know if this characteristics change when the initial situation varies: numerical data and contextualized information.

1.1. Problem posing

The literature contains different ways of interpreting the expression *Problem Posing* (in lowercase from now on), depending on the perspective from which it is implemented in the classroom or analyzed in the research. Cai and Hwang (2020), building on the definition forwarded by Cai et al. (2020), they proposed the following:

> *"By problem posing in mathematics education, we refer to several related types of activity that entail or support teachers and students formulating (or reformulating) and expressing a problem or task based on a particular context (which we refer to as the problem context or problem situation)."* (p. 2).

A problem-posing activity can be approached in different ways. Stoyanova and Ellerton (1996) distinguish between free, semi-structured, and structured problem-posing tasks. A problem-posing task will be referred to as free "*when students are asked to generate a problem from a given, contrived or naturalistic situation. Some directions may be given to prompt certain specific actions*" (p. 519); as semi-structured "*when students are given an open situation and are invited to explore the structure and to complete it by applying knowledge, skills, concepts and relationships from their previous mathematical experiences*"; and as structured "*when problem-posing activities are based on a specific problem*" (p. 520).

Elsewhere, Cai et al. (2022) distinguish between two parts of the problem-posing task: the *situation* and the *prompt*. First, the *situation* is considered to provide the basis and the starting point for posing the problems. It provides the context and/or the data, that is, the initial information. Second, the type of problem posing requested is considered, which is called the *prompt*; that is, specifically lets posers know what they are expected to do. Depending on the goals set, several prompts can be proposed for the same situation (Table 1) and vice versa, that is, several situations can be proposed with the same prompt.

In the literature on problem posing, there are studies whose focus is to understand the nature of problem posing itself, including examining and evaluating the types, quality, complexity and quantity of posed problems (e.g. Kwek, 2015; Silber & Cai, 2021; Yao et al., 2021), as well as the competencies, strategies, skills, and other factors that allow for productive problem posing (e.g. Ellerton, 2013; Leikin & Elgrably, 2020).

In those studies, researchers have proposed to analyze certain characteristics of the posed problems. One of that is the plausibility which allows us to assess the solvability of the problems posed. To examine the arithmetic problem posing behaviors of sixty-three prospective elementary school teachers, Leung and Silver (1997) classified, first of all with a three-step process, the problems as mathematical or non-mathematical, as plausible or non plausible, and as containing sufficient or insufficient information. In this study, a non-mathematical problem is defined as one that can be solved without mathematics; a plausible problem is "*feasible and no discrepant information could be found*" in it; and a problem contains sufficient information if it is "*solvable using information found in the task stem and/or in the response itself*" (Leung & Silver, 1997, p. 10). The authors state that these features allow us to get information about the quality of the posed problems. Moreover, this study analyze the posed-problem complexity according to the number of arithmetic steps needed to solve them.

Cankoy (2014) conducted a study focused on the contributions of an instructional approach to the quality of the problems posed by fifth grade students in free structured situations. In this case, problems were classified in terms of mathematical solvability, reasonability and mathematical structure. For the first category, two subcategories were considered: a problem is solvable if "*the information given in the problem is sufficient to solve the problem and find the answer*" and it is unsolvable if the "*information given in the problem is not sufficient to solve the problem and find the answer*" (Cankoy, 2014, p. 221). Therefore, the first category combines the categories plausibility and sufficiency of information considered by Leung and Silver (1997). For the second category, Cankoy (2014) regards a problem as reasonable when "*the information given in the problem and the answer found are real or practical in real life*" and unreasonable if "*the information given in the problem and the answer found are unrealistic or not practical in real life*" (p. 221). Finally, in the analysis of the mathematical structure, it is distinguished between problems with a result-unknown "*which have a mathematical structure in which the*

Table 1
Examples of situation and prompts in problem posing (Cai et al., 2022).

Situation	ABC is an equilateral triangle. D, E, and F are midpoints of the sides of DABC. Show that the area of DDEF is ¼ the area of DABC.
Prompt 1	Based on the above problem, use the "what if not" strategy to pose two mathematical problems.
Prompt 2	Based on the above problem, use the "what if not" strategy to pose as many mathematical problems as you can.

result is unknown" and problems with a start-unknown "*which start with unknown(s)*" (Cankoy, 2014, p. 221).

As in Leung and Silver (1997), Grundmeier (2015) considered some characteristics to describe, in a study with 19 prospective elementary and middle school teachers, changes of participant's posed problems as they gained problem-posing experience. For that, four categories were defined taking into account plausibility, sufficiency of information and complexity. So, this author distinguished between problems that are *non-plausible* (*NP*), if it contains invalid statements and is not solvable, even when more information is added; *plausible with insufficient information* (*P1*), if it can be solved even if the statement implies or does not make explicit part of the information; *plausible with sufficient information on one mathematical task* (*P2*) *or several mathematical tasks* (*P3*), depending on the number of steps needed to solve it.

In a recent study, Leavy and Hourigan (2022b) have proposed a framework for posing elementary Mathematics problems which describes desirable features for those problems. The framework consists of eight indicators each representing a characteristic of a quality mathematics problems such as use of a motivating and engaging context, clarity in language and cultural context, curriculum coherence, etc. Some indicators are closely related with the features analyzed in previous studies mentioned before. The use of a motivating and engaging context in a problem permits establishing connections between mathematics and the real world, so, in this line, if the problem has developed this characteristic, it would be reasonable in the sense of Cankoy (2014). The indicator referring to the use of an appropriate number of solution steps to support reasoning is related with the feature considered by Leung and Silver (1997) for the arithmetic complexity and by Grundmeier (2015) for the categories P2 and P3, but, in this case, the authors highlight that, although problems requiring two steps or more usually have higher levels of cognitive demand and require greater mathematical proficiency levels, in this indicator should consider the type of mathematical reasoning involved and not only the number of steps.

As products may be more accessible by analysis than processes, most problem-posing studies focus on posed problems (Baumanns & Rott, 2022). Nevertheless, some researchers have identified general strategies that students use to pose problems (e.g., Baumanns & Rott, 2022; Cai & Cifarelli, 2005; English, 1998; Koichu, 2020).

Other authors have studied some task variables that can influence the problems proposed by students (Leung & Silver, 1997; Silber & Cai, 2017; Zhang et al., 2022). A task variable is defined as "any characteristic of problem tasks which assumes a particular value from a set of possible values" (Kulm 1979, p. 1, in Golding & McClintock, 1979). Thus, a task variable might be the quantity of numerical information initially given, the starting context, the format of the situation, the type of prompt, and so on. Against this background, Leung and Silver (1997) conducted a test of arithmetic problem posing with prospective elementary school teachers. In their study, they found that most subjects were able to pose solvable and complex problems, and that problem-posing performance was better when the task contained specific numerical information than when it did not. In their study, Silber and Cai (2017) found that while the task format (if it is free or structured problem-posing situation) had limited impact on the complexity of problems posed, pre-service teachers in the structured-posing condition may have more closely attended to the mathematical concepts in each task, and may have also impacted their process of posing problems than those in the free posing condition. In a recent study with elementary school students, Zang et al. (2022) determined that students performed significantly better on the task with context than on the task without context, and that the students were generally more successful on the problem-posing test that included specific numerical information than on that which did not. Another result of this study indicates that, in a structured task, students who were able to solve the problem correctly were able to propose more solvable mathematical problems than those students who were not able to solve it correctly.

Despite the previous results, it is necessary to continue delving into the impact that certain task variables have on the problems formulated by future teachers (e.g. the type of information given from which they have to pose problems).

1.2. Teacher education and problem posing

Kiliç (2015) notes that "problem posing is an effective mathematical activity that can help people to construct mathematical knowledge through integrating their existing structures of knowledge" (p. 772). In that same sense, Tichá and Hošpesová (2009) indicate that problem posing contributes to the development of mathematics knowledge during the pre-service education of primary school teachers. Although little is known about how teachers integrate problem posing into mathematics teaching (Cai & Hwang, 2020), research has shown that only if pre-service or in-service teachers gain experience by developing problem-posing activities, they will be able to incorporate it into their practice and promote it among their students (Singer et al., 2013). It is therefore necessary to develop specific educational programs that give teachers the knowledge to effectively use problem posing in their classrooms (Cai & Hwang, 2020).

Findings from research indicate that the problems created by pre-service and in-service teachers with no prior and explicit education in this area have not a high mathematical quality (Cai et al., 2015); most of them are arithmetic, can be solved just with one step and have a unique solution or they present some ambiguity in their formulation or some mathematical mistake that makes them unsolvable (e.g. Chapman, 2012; Crespo, 2003; Leavy & Hourigan, 2020). Studies also have revealed a lack of concern for pre-service teachers to establish a reasonable connection with the formulated problem and the real world or a meaningful context (Leavy & Hourigan, 2020; Silver & Burkett, 1993; Simon, 1993).

A strategy that has shown positive results in developing the ability to formulate problems in future teachers is *letter writing* (e.g. Crespo, 2003; Leavy & Hourigan, 2022a; Norton & Kastberg, 2012). This method consists of a letter-writing exchange between pre-service teachers and students of any grade, where the pre-service teacher poses a task to the student, analyzes their response and gives feedback to their ideas (Phillips & Crespo, 1996). Research results indicate that pre-service teachers pose more problems with multiple approaches and solutions that are cognitively more complex after participating in a letter writing exchange with primary school students (Crespo, 2003; Leavy & Hourigan, 2022a). *Letter-writing* has also been used with high school pre-service teachers

(Norton & Kastberg, 2012). In this research, the analysis of students' responses by pre-service teachers helped them to pose problems with a greater cognitive demand.

Some studies show that problem posing can be an appropriate way to bring pre-service teachers closer to the reality of their profession, for example, to be aware of the multidimensional nature of a mathematical problem (Tichá & Hošpesová, 2013) or to develop their understanding of what constitutes a good problem (Leavy & Hourigan, 2022b). Even more, whether the discussion about problems is carried out among peers or with school students influences the way in which pre-service teachers perceive the interest of difficulty of the problems posed (Guberman & Leikin, 2013).

Another relevant research result is that pre-service teachers create better problems when given prior information, for example, when starting from given images or a numerical data already presented (Crespo, 2003; Leung & Silver, 1997). They were also observed to be more successful when reformulating problems already given than when they have to pose them with no prior information (Stickles, 2011). Our research seeks to analyze what differences may appear when different initial problem posing situations are used. In particular, will the problems formulated by future teachers have different characteristics if the initial information they are given is a numerical value or a context?

1.3. Fractions and problem posing

Whenever we pose problem involving fractions, conceptual and procedural knowledge is activated. Specifically, if the problem is contextualized within a real-world situation, fractions may show some of the meanings outlined below (Behr et al., 1993):

- Part-whole: occurs in situations in which a whole (continuous or discrete) is divided into equivalent parts. The whole is designated as the unit, and the fraction expresses the relationship between the number of parts and the total number of parts into which the whole has been divided.
- Measure: consists of using a unit fraction repeatedly to determine the distance from a starting point.
- Division (quotient): occurs in problems associated with the operation of dividing one whole number by another.
- Ratio: the fractions provide a comparative index between two quantities or sets of units.
- Operator: the fraction is interpreted as something that acts on and modifies a situation, that is, it assumes a transformative role through a multiplication or division operation.

An aspect to consider in teacher education is achieving an appropriate use of the different meanings of fractions. Lamon (2012) points out that instruction that focuses only on the part-whole meaning (which is often the most common) leads to a weak conceptual understanding. Besides, Behr et al. (1997) indicated that prospective teachers show a deeper understanding of the part-whole meaning compared to other meanings. On the other hand, in primary education, problems involving fractions are posed with different mathematical structures, among which are distinguished the following:

- Concept: Expressing situations from a context or a graphical representation using fractions, without setting up arithmetic operations. For example, at a birthday party, there are 15 children. When distributing candies among three flavors, 6 children choose chocolate, 4 children choose strawberry, and 5 children choose vanilla. What fraction of children chooses each of the flavors?"
- Order: Ordering fractions. For example, in the previous problem, represent the preferences for each flavor in fractions with respect to the total number of children and ask to order them from least to greatest preference.
- Additive operation: perform an addition or subtraction operation.
- Multiplicative operation: perform a multiplication or division operation.

Clearly, problems involving multiple steps can be created, in which the previous structures are combined.

Some studies have indicated that fractions cause difficulties for future teachers (Olanoff et al., 2014; Tichá & Hošpesová, 2013). For instance, Olanoff et al. (2014) present an extensive review of 43 research on the mathematical content knowledge of prospective elementary school teachers regarding fractions. The reviewed papers concur in stating that prospective teachers' knowledge of fractions is relatively deep when it comes to performing operations, but weaker in relation to understand the meanings or explanations of why the operations with fractions work. In this sense, Tichá and Hošpesová (2013) pointed out that the disconnection between the conceptual and procedural knowledge is reflected in their limited ability to relate fraction operations to corresponding graphical representations or use of manipulative models. Recently, in a study with 79 prospective teachers about modeling fraction addition (area models, length models, and discrete sets), it was observed that despite solving the operation correctly, many of them did not represent it accurately using the different models (Lee & Lee, 2023).

Olanoff et al. (2014) indicate that in most studies conducted with prospective teachers, there is a lack of providing ways to help improve their knowledge of fractions. In this context, some studies show that learning how to pose problems can contribute to enhancing the understanding of the meanings and mathematical structures of fractions; however, it is not an activity without challenges. Thus, in the case of fractions Ma (1999) compared the ability of primary school teachers in China and the United States (US) to create fraction-division problems. She concluded that US teachers had difficulty producing appropriate problems and showed inadequate conceptions of fractions, while those in China posed at least one problem based on different meanings of fraction. In their study with pre-service elementary teachers, Xie and Masingila (2017) noted the difficulties both solving and posing problems with fractions, which they associated with a lack of experience posing problems and with a poor understanding of fractions and their operations. Finally, Kiliç (2015) conducted a study with 90 pre-service elementary teachers in which she asked them to create problems using the

Table 2
Characteristics of the activities proposed.

	Prompt	Situation
PP-Num	Formulate three fraction problems of varying difficulty	Given these numbers: 1/4 and 3/8. Use them as a given or solution.
PP-Context	Formulate three fraction problems of varying difficulty	Initial context with a whole number: *18 students take part in an end-of-year trip. The teacher asks what places they would like to travel to.*

fractions ½ and/or ¾, finding that the problems they proposed mainly involved addition and multiplication in symbolic contexts, more so than contextualized, and mostly a combination of two operators.

We propose delving into the characteristics of the problems posed as a response to two problem-posing tasks that, despite sharing a prompt, start from two different situations. In addition, these tasks were carried out with pre-service primary education teachers, meaning the results of the study will help us know how their educational program has developed this professional competence. For this analysis, we will study the following characteristics in each problem: plausibility (or solvability), reasonability of the information given in the statement in contrast to the realistic or practical in real life, meaning of fraction and mathematical structure of the problems posed.

To this end, we considered the following research questions:

1. In what way do they initially provided problem-posing task situations influence the plausibility of the problems posed by pre-service teachers?
2. And, on the plausible problems posed:
   a. Are they reasonable from the contextual point of view? What differences may arise for each given situation proposed?
   b. What meanings and mathematical structures do they include? What differences may arise for each given situation proposed?
   c. Can profiles be established for pre-service teachers given the problems posed and their plausibility? Do these profiles vary according to the situation?

2. Methodology

2.1. Participants

The research was conducted with 205 students in their third year of a Bachelor's Degree in Primary Education Teacher in Spain. This Bachelor' program just includes three compulsory subjects directly related to mathematics, its learning and teaching. First one, called *Mathematics,* takes place during the second year, whose focus is to deepen curricular mathematical contents in primary school. At the time of the research, the participants were taking the second subject, called *Didactics of Numbering, Statistics and Randomness* course. During those subjects, students do not receive any train in problem posing. Participation in the study was voluntary.

2.2. Data

The data were collected from a questionnaire with three different problem-posing tasks, two of them with the same prompt and the third with a different prompt. In this paper, we focus on the analysis of the first two tasks with the same prompt (Table 2) and different starting situations: a numerical situation (PP-Num) and a contextual situation (PP-Context). One of our goals is to observe how the initial situation, which provides the information for the task, affects the plausability of the mathematical problems posed.

The instructions given for formulating the problems, that is, the prompt, proposed posing three problems of varying difficulty in order to challenge the students to mobilize their mathematical knowledge of fractions. They had to use only pencil and paper, they could add all the information they wanted, and it was not necessary to solve the problems. This was stipulated to keep the students from rejecting problems they were able to formulate but not solve. The participants had a maximum of 15 min to answer each task.

The situations presented to them in these two activities are different and comprise the variables of the task to be analyzed. Thus, in the first task (Table 3), the situation consisted only of numerical information provided to the students, which comprised two fractions of the same family. This situation is analogous to that presented in Tichá and Hošpesová (2013) and Kilic (2015). In this situation, fractions as part of a whole are given, so in the problem they should establish the whole where they came from.

In the second task (Table 4), the situation consists of an initial context, and the number of children may be regarded as the unit to be fractioned.

Some students failed to formulate the three problems that were asked of them. In task 1 (PP-Num), among all the participants a total of 603 problems were posed, while in task 2 (PP-Context), they posed 614 problems (Table 5). Therefore, the data to be analyzed consist of a total of 1217 problems posed by 205 pre-service Primary Education teachers.

2.3. Data analysis process

Two different processes were performed to analyze the data, taking into account the nature of the research questions. On the one hand, each problem posed by the pre-service teachers was analyzed (research objective 1, 2a and 2b) and, on the other, an analysis was done by subject (research objective 2c).

For each of the 1217 problems posed by the participants, the first step was to determine if it involved fractions or not. If it did not (*NF*), it was not encoded any further. If it did, the characteristics of *plausibility, reasonability, mathematical structure* and *meanings of*

Table 3
Task 1.

Pose three problems of varying difficulty that use the numbers 1/4 and 3/8. Either of these numbers may be a given or the solution. Remember that you can add any type of information (numerical, contextual, etc.).

Table 4
Task 2.

Pose three problems of varying difficulty that involve fractions and whose initial context is as follows:
18 students are going on an end-of-year trip. The teacher asks them where they would like to travel.
Remember that you can add any type of information (numeric, contextual), provided the information given in the box is unchanged.

Table 5
Number (percentage) of students who posed 1, 2 or 3 problems for each task proposed.

Task	1 problem	2 problems	3 problems	Total
PP-Num	1 (0.4)	10 (4.8)	194 (94.6)	205
PP-Context	0 (0)	1 (0.4)	204 (99.5)	205

fraction were analyzed. For each of these characteristics, a series of categories was established (Table 6).

For the plausibility, the four categories have been considered by the classification of Grundmeier (2015), which include the features of solvability, sufficiency of information and complexity (Section 1.1).

The reasonability (Cankoy, 2014) was analyzed only in problems contextualized in real life situations in keeping with Cankoy's definition.

For the mathematical structure, which refers to the type of relationships or operations that are involved in solving the problem, it was analyzed the followings (Section 1.3): *concept*, *order*, *additive*, *multiplicative*, or a *combination thereof*.

Finally, only in contextualized problems, we also analyzed the meanings of fraction in the statement of the problem, taking into account that each problem posed can exhibit more than one meaning (Section 1.3): *Part-whole, Measure, Division, Ratio, Operator.*

It was followed a method of analysis of blind multiple coding process involving four coders trained for this purpose. Each of the coders independently logged the code for each of the characteristics analyzed (Table 6) and for each problem in a Google form. So each problem was coded by the four independent coders. Mismatched encodings were first discussed between the four coders and, if they did not reach a deal, the cases were analyzed for the research team (the authors of this paper) to reach a consensus.

The responses to the encoding were stored directly in a Google spreadsheet associated with the form. Once all the problems were coded, the frequency of the categories defined for each feature studied was analyzed, and for each of the two tasks (PP-Num and PP-Context), in order to answer research questions 1, 2a and 2b.

To answer research question 2c, a performance analysis was done on each task per subject. To do this, we used the information about the plausibility of the set of problems posed by each of the 205 participants and established a set of profiles. We defined the profiles according to the maximum number of plausible problems with sufficient information (*P2* or *P3*) that the participants formulated (Table 7). Thus, profile A corresponds to participants who posed three P2 or P3 problems; in profile B/C are those who formulate two P2 or P3 problems; profile D/E/F is formed by participants that posed one P2 or P3; and profile G/H/I/J for those that do not formulate any P2 or P3 problem.

The subjects' performance was then compared by indicating their profile in each of the two tasks, in order to identify any differences between the tasks.

3. Analysis of the results

Each category observed is analyzed in the sections below.

3.1. Plausibility

The global analysis of the 1217 problems posed by the participants (Table 8) shows that 21 problems (1.7%) are problems whose solution does not require the use of fractions (*NF*), and 94 problems (7.7%) have mathematical errors that make them unsolvable (*NP*).

Table 6
Analysis categories for the problems posed.

Characteristics	Categories
Mathematical content	*NF:* Fractions are not necessary to solve the problem.
Plausibility and sufficiency of the data	*NP:* Not plausible, with mathematical errors. *P1:* Plausible with insufficient information to solve it. *P2:* Plausible with sufficient information, posing a single mathematical task. *P3:* Plausible with sufficient information, posing multiple mathematical tasks.
Reasonability	*Reasonable, not reasonable.*
Mathematical structure	*Concept, Order, Additive, Multiplicative or a combination thereof.*
Meaning of fraction	*Part-Whole, Measure, Division, Ratio, Operator or a combination thereof.*

Table 7
Profiles of subjects based on the number and type of plausible problems posed in the activity.

Profile	Plausibility		
	NF/NP	P1	P2/P3
A	0	0	3
B	0	1	2
C	1	0	2
D	0	2	1
E	1	1	1
F	2	0	1
G	0	3	0
H	1	2	0
I	2	1	0
J	3	0	0

As an example, in Fig. 1 there is a problem that could be solved by adding two natural numbers, while in Fig. 2 there is a mathematical mistake because fractions in the problem refer to the same unit, but the solution requires that each fraction refer to a different unit.

Related with the other categories, a total of 180 (14.8%) problems were formulated with insufficient information (*P1*), the majority of which do not indicate that fractions are referred to the same unit (Fig. 3).

Most of the problems posed by the participants were plausible with sufficient data solvable with a unique mathematical task (*P2*) or more than one (*P3*), 459 problems (37.7%) and 463 problems (38%) respectively. As an example, Fig. 4 shows a problem which solution is only calculating the fraction of a total (*P2*); while Fig. 5 shows an example with two mathematical tasks (*P3*), namely calculating the parts and determining the rest of the students.

Comparing the PP-Num and PP-Context tasks, there are no differences between the results of the *NF* and *NP* categories. Practically 90% of the problems posed in both tasks are plausible (Table 8), although 24.7% of the problems in the PP-Num activity lack information (*P1*), many more than the 5% in the same category in the PP-Context task. By contrast, the PP-Context activity has about 16% more problems in the *P3* category than in the same category in the PP-Num activity.

A more detailed analysis of the plausible problems with lack of information (*P1*) in the PP-Num activity (Table 9) shows that most of them either do not offer the total amount to which the fractions apply (*total quantity*), or when they give fractions that originate from different units (two pizzas, two plots), they do not indicate that these units have the same area (*indicate equality*) or surface. It is clear that these problems ignore the fact that fractions must refer to a common unit in order to operate with or compare them.

3.2. Reasonability

As we explain in the methodology, reasonability was only analyzed for plausible (*P1, P2 and P3*) and contextualized problems. In the PP-Num task, 496 were contextualized problems (91.5%); while in the PP-Context activity, all the problems posed rely on daily life contexts (Table 10). The data indicate that the participants posed, in total, 850 reasonable problems, with the percentages being similar in both tasks: 76.2% and 78% in PP-Num and PP-Context, respectively.

One of the aspects that made problems unreasonable contextually had its origin in using non-integer numerical values that do not make sense in the context. For example, in Fig. 6, the number of students who selected one country is not a whole number.

Another unreasonable aspect involves the use of quantities that are either very large or very small for the context used. For example, in Fig. 7, the problem is contextualized in a classroom with 380 students, which may be reasonable from a numerical point of view, but not realistic in a school.

3.3. Mathematical structure of the problems posed

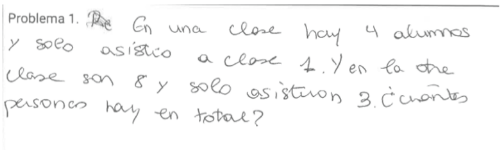
The analysis of the mathematical structure of the problems posed helps us learn how the students use mathematical relationships and operations with fractions. We found problems with the four structures mentioned in Section 1.3 (Table 11): *concept*, *order*, *additive* and *multiplicative*.

As an example, it is shown a problem posed of each structure in following figures, from number 8 to 11.

We observed differences in the various structures between the two situations analyzed (Table 11), although most are problems in which several structures are combined.

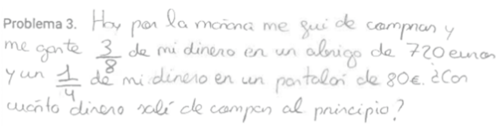
Table 8
Plausibility of the problems. Number (percentage) of problems in each category.

	NF	Not plausible	P1	P2	P3	Total
PP-Num	14 (2.3)	47 (7.8)	149 (24.7)	212 (35.2)	181 (30)	603
PP-Context	7 (1.1)	47 (7.7)	31 (5)	247 (40.2)	282 (46)	614
Total	21 (1.7)	94 (7.7)	180 (14.8)	459 (37.7%)	463 (38%)	1217


Problema 1. En una clase hay 4 alumnos y solo asistió a clase 1. Y en la otra clase son 8 y solo asistieron 3. ¿Cuántas personas hay en total?

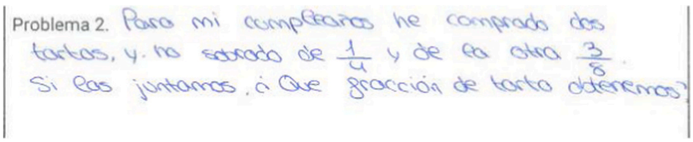
Problem 1. In one class there are 4 students and only one is present. In another class there are 8 students and only 3 are present. How many people are there in total?

Fig. 1. *Example of a problem in the NF category (Student-118).*


Problema 3. Hoy por la mañana me fui de compras y me gaste $\frac{3}{8}$ de mi dinero en un abrigo de 720 euros y un $\frac{1}{4}$ de mi dinero en un pantalón de 80€. ¿Con cuánto dinero salí de compras al principio?

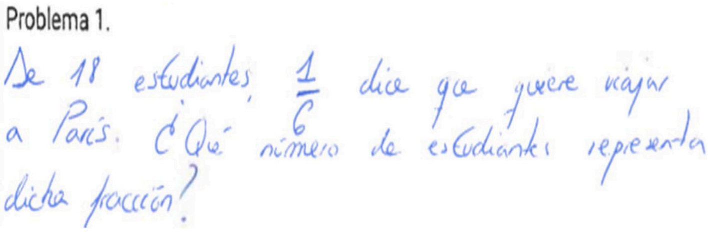
Problem 3. This morning, I went shopping and I spent 3/8 of my money on a coat costing €720, and ¼ of my money on a pair of trousers costing €80. How much money did I have when I went shopping this morning?

Fig. 2. *Example of a problem in the NP category (Student-157).*


Problema 2. Para mi cumpleaños he comprado dos tartas, y ha sobrado de $\frac{1}{4}$ y de la otra $\frac{3}{8}$. Si las juntamos, ¿Que fracción de tarta obtenemos?

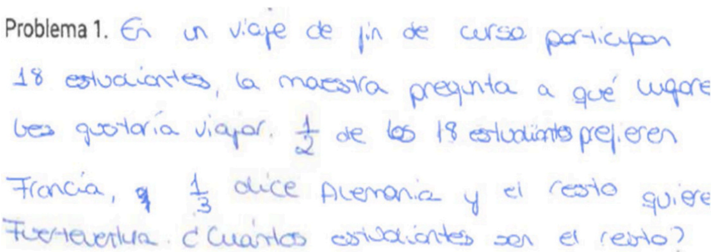
Problem 2. For my birthday I bought two cakes and there is 1/4 left of one cake and 3/8 left of the other. If we join the parts, which fraction of the cakes do we have?

Fig. 3. *Example of a problem in the P1 category (Student-20).*


Problema 1.
De 18 estudiantes, $\frac{1}{6}$ dice que quiere viajar a París. ¿Qué número de estudiantes representa dicha fracción?

Problem 1. There are 18 students. 1/6 say they want to travel to Paris. What number of students does this fraction represent?

Fig. 4. *Example of a problem in the P2 category (Student-181).*


Problema 1. En un viaje de fin de curso participan 18 estudiantes, la maestra pregunta a qué lugares les gustaría viajar. $\frac{1}{2}$ de los 18 estudiantes prefieren Francia, $\frac{1}{3}$ dice Alemania y el resto quiere Fuerteventura. ¿Cuántos estudiantes son el resto?

Problem 1. 18 students are going on the end-of-course trip. The teacher asks what places they would like to travel to. 1/2 of the 18 students prefer France, 1/3 say Germany and the rest want Fuerteventura. How many students make up the rest?

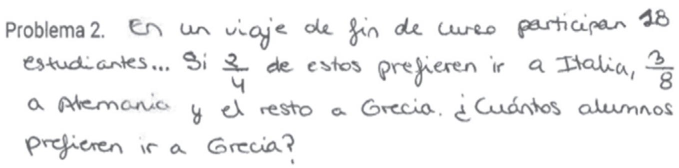
Fig. 5. *Example of a problem in the P3 category (Student-202).*

Table 9
More frequent errors in the P1 problems.

Activity	Total quantity	Indicate equality	Combination of above	Other	Total
PP-Num	49 (32.9)	48 (32.2)	3 (2)	49 (32.9)	149

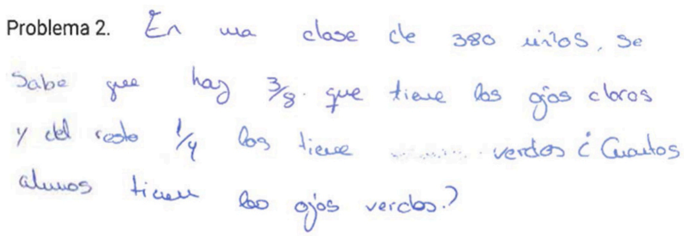
Table 10
Context and reasonability of the P1, P2 and P3 problems.

	Not contextualized	Contextualized		Total
		Reasonable	Unreasonable	
PP-Num	46 (8.5)	413 (76.2)	83 (15.3)	542
PP-Context	0 (0)	437 (78)	123 (22)	560
Total	46 (4.2)	850 (77.1)	206 (18.7)	1102


Problema 2. En un viaje de fin de curso participan 18 estudiantes... Si 2/4 de estos prefieren ir a Italia, 3/8 a Alemania y el resto a Grecia. ¿Cuántos alumnos prefieren ir a Grecia?

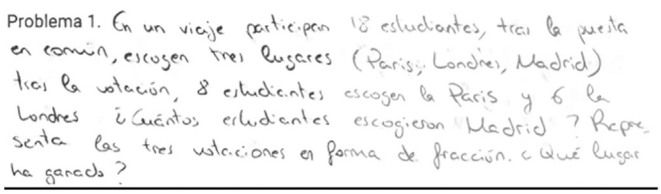
Problem 2. 18 students are going on the end-of-course trip. If 2/4 prefer Italy, 3/8 Germany and the rest Greece, how many students prefer Greece?

Fig. 6. *Example of unreasonable problem due to decimal amount (Student-67).*


Problema 2. En una clase de 380 niños, se sabe que hay 3/8 que tiene los ojos claros y del resto 1/4 los tiene verdes ¿Cuantos alumnos tienen los ojos verdes?

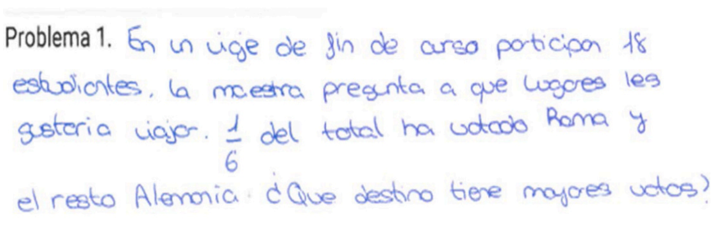
Problem 2. In a class of 380 children, 3/8 of them have light eyes and 1/4 of the rest have green eyes. How many students have green eyes?

Fig. 7. *Example of unreasonable problem due to excess quantity (Student-93).*


Problema 1. En un viaje participan 18 estudiantes, tras la puesta en común, escogen tres lugares (París, Londres, Madrid) tras la votación, 8 estudiantes escogen la Paris y 6 la Londres ¿Cuántos estudiantes escogieron Madrid? Representa las tres votaciones en forma de fracción. ¿Qué lugar ha ganado?

Problem 1. 18 students are going on the end-of-course trip. After discussing it, they choose three destinations (Paris, London, Madrid). After voting, 8 students choose Paris and 6 London. How many students chose Madrid? Represent the three votes in the form of a fraction. Which place won?

Fig. 8. *Example of concept structure problem (Student-103).*


Problema 1. En un viaje de fin de curso participan 18 estudiantes, la maestra pregunta a que lugares les gustaría viajar. 1/6 del total ha votado Roma y el resto Alemania. ¿Que destino tiene mayores votos?

Problem 1. 18 students are going on the end-of-course trip. The teacher asks them what places they would like to travel to. 1/6 of the total voted for Rome and the rest for Germany. Which destination has the most votes?

Fig. 9. *Example of order structure problem (Student-18).*

Table 11
Number (percentage) of problems classified by mathematical structure.

Activity	Concept	Order	Additive	Multiplicative	Combined	Total
PP-Num	64 (11.8)	36 (6.7)	118 (21.8)	65 (12.0)	259 (47.7)	542
PP-Context	68 (12.1)	8 (1.4)	72 (12.9)	113 (20.2)	299 (53.4)	560

When the problem posed only has one structure, in the PP-Num activity the *additive* structure is more frequent (e.g. Fig. 10), while in PP-Context the *multiplicative* structure is more often (e.g. Fig. 4).

However, in the problems that combine two or more structures (Table 12), the *additive-multiplicative* combination is the most frequent in both activities, accounting for more than 75% of the problems posed with two or more structures in the PP-Num activity, and more than 50% for PP-Context.

3.4. Meaning of fraction

Fractions convey multiple meanings (Section 1.3), which makes the problem-solving activity more mathematically enriching.

The two most frequent meanings in the posed problems were *part-whole* (e.g. Fig. 10) and *operator* (e.g. Fig. 7). In each activity, there is more than a 35% of problems where these meanings are used (Table 13). *Division* (Fig. 11) and *ratio* problems are very infrequent, as is the meaning of *measure*, which only appears sparsely in combination with other meanings.

The final results of the classification are shown in Table 13. We find that these are consistent with the meanings of fraction that are usually presented in the classroom, in which the *division, measure or ratio* meanings are uncommon.

In problems that combine more than one meaning, the *part-whole-operator* combination prevails in both tasks, occurring in 53 PP-Num problems (9.7%) and 34 PP-Context problems (6.0%). There are thus no relevant differences in each situation in the significance of the use of fraction. It is worth noting that students rarely resort to giving other meanings in the two tasks.

3.5. Problem-posing student profiles

Next, we will consider the performance of each pre-service teacher in the two activities proposed. We will do so by establishing student profiles based on the number of problems posed with all the necessary information, whether they have one or two mathematical tasks (*P2* or *P3*) (Table 7). Our intention is to determine each student's level of achievement when posing fraction problems in relation to plausibility and according to the situation posed.

Ten subject profiles were identified based on the plausibility of the three problems posed, as well as those subjects who did not complete the activity with three problems (*NC*), and thus did not satisfy the prompt of the problem-posing task (Table 2).

Table 14 shows that in PP-Context activity, twice as many students were able to pose three plausible problems as in PP-Num activity. The students posed less plausible problems (profiles G, H, I and J) in PP-Num activity. This is consistent with what is expected when working in a context that proposes distributing a whole unit (students in a class) into preference groups (countries to visit).

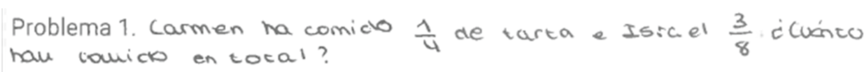
Table 15 shows the students' performance in the two tasks: PP-Num and PP-Context. The diagonal in the table shows that a total of 53 students maintained their profile in the two activities. Of note is the low number of pre-service teachers who are in profile A in both tasks (18.5%).

It should be noted that in the PP-Context activity, 54.6% (112) of the students improved their performance compared to the PP-Num activity (lower triangular matrix in Table 15). It is especially relevant that 16 subjects (7.8%) who did not offer any plausible problems in the PP-Num task (G/H/I/J-profiles) posed two or three plausible fraction problems in the PP-Context activity, although one-third of the A-profile subjects in PP-Num posed at least one implausible problem in PP-Context (23 students of 61 A-profile).

4. Conclusions

In this paper, we have studied how pre-service primary school teachers who are finishing their initial training pose mathematical problems with fractions. To do this, they were asked to engage in two problem-posing tasks based on two different initial situations (free use of two given fractions and given context associated with a whole amount) with the same prompt: pose three problems with fractions of varying difficulty.

To answer the first research question, we proceeded analyzing how the situation given affects the plausibility of the problems posed (Grundmeier, 2015). The data obtained indicate that in both situations, the pre-service teachers were able to pose plausible problems with fractions with a high level of effectiveness and no notable differences between them. If we focus on the number of problems posed



Problem 1. Carmen has eaten ¼ of a cake and Israel ⅜. How much have they eaten in total?

Fig. 10. *Example of additive structure problem (Student-35).*

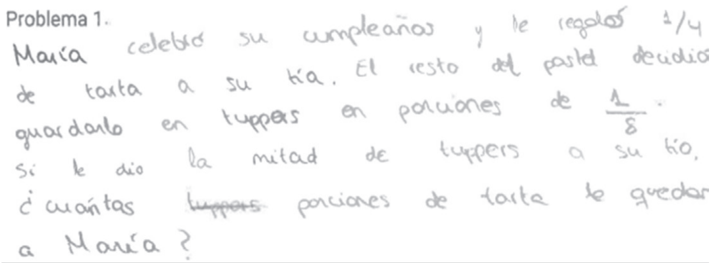
Table 12
Detail of the results of the combination of mathematical structures in the problems posed.

Activity	Concept order	Concept additive	Concept Multip.	Order additive	Order Multip.	Additive Multip.	Comb. >2	Total
PP-Num	8 (3.1)	18 (7.0)	5 (2.0)	8 (3.1)	6 (2.3)	198 (76.4)	16 (6.1)	259
PP-Context	3 (1.0)	45 (15.1)	12 (4.0)	-	33 (11.0)	158 (52.8)	48 (16.1)	299

Table 13
Problems according to the meaning of the fraction.

	Not contextualized	Contextualized						
		P-W	Div	Meas	Oper	Ratio	Comb. >1	Total
PP-Num	46 (8.5)	222 (41.0)	7 (1.3)	0	198 (36.5)	3 (0.5)	66 (12.2)	542
PP-Context	0	199 (35.5)	1 (0.2)	0	325 (58.0)	0	35 (6.3)	560

Legend: P-W = Part-Whole; Div = Division; Meas = Measure; Oper. = Operator; Ratio; Comb. > 1 = more than one meaning.


Problema 1.
Maria celebró su cumpleaños y le regaló 1/4 de tarta a su tía. El resto del pastel decidió guardarlo en tuppers en porciones de 1/8. Si le dio la mitad de tuppers a su tío, ¿cuántas ~~tuppers~~ porciones de tarta le quedar a María?

Problem 1. Maria celebrated her birthday and gave 1/4 of a cake to her aunt. She decided to store the rest of the cake in containers in 1/8 portions. If she gave half the containers to her aunt, how many servings of cake does Maria have left?

Fig. 11. *Example of meaning of fraction problem (division) (Student-91).*

Table 14
Number (percentage) of students in each profile based on the number and type of plausible problems posed.

Profile	Plausibility			PP-Num	PP-Context
	NF/NP	P1	P2/P3		
A	0	0	3	61 (29.8)	133 (64.9)
B	0	1	2	56 (27.3)	25 (12.2)
C	1	0	2	23 (11.2)	34 (16.6)
D	0	2	1	24 (11.7)	2 (1)
E	1	1	1	13 (6.3)	2 (1)
F	2	0	1	1 (0.5)	7 (3.4)
G	0	3	0	2 (1)	0 (0)
H	1	2	0	9 (4.4)	0 (0)
I	2	1	0	4 (2)	0 (0)
J	3	0	0	1 (0.5)	1 (0.5)
NC	-	-	-	11 (5.3)	1 (0.5)
			TOTAL	**205**	**205**

Table 15
Performance of student profiles in the two tasks.

PP_Num / PP-Context	A	B/C	D/E/F	G/H/I/J	NO	Total
A	38 (18.5)	21 (10.3)	2 (1.0)	-	-	61 (29.8)
B/C	62 (30.2)	11 (5.4)	4 (2.0)	1 (0.4)	1 (0.4)	79 (38.4)
D/E/F	18 (8.8)	16 (7.8)	4 (2.0)	-	-	38 (18.6)
G/H/I/J	9 (4.4)	7 (3.4)	-	-	-	16 (7.8)
NC	6 (3.0)	4 (2.0)	-	1 (0.4)	-	11 (5.4)
Total	133 (64.9)	59 (28.9)	10 (5)	2 (0.8)	1 (0.4)	**205**

that contain all the information needed to solve them (*P2* and *P3*), we see better results when the initial situation provides the context on which to base the problem (PP-Context), versus the case in which only numerical information is provided (PP-Num). The main absence of information in the PP-Num activity refers to the fact that in the statement of the posed problem no indication is given that the parts must be equal, or that when comparing or adding two fractions, the starting unit must be the same. The pre-service teacher takes these aspects as a "given". This is manifested in the second situation (PP-Context), since the initial situation provided the total, and the parts comprise a given number of children (with no variations in terms of the size). In short, the initial situation in the PP-Num task reveals a certain conceptual misunderstanding that is relevant to the meaning of fraction. This could persist into their professional work as in-service teachers with primary school students.

We then turned our interest to determining certain characteristics of the plausible problems (*P1*, *P2*, and *P3*), and to analyzing potential differences among the situation involving the reasonability, problem mathematical structure and meaning given to fraction. With respect to the reasonability, no differences were observed between the two situations offered since the pre-service teachers constructed reasonable problems for the given context, although a significant number of them provided answers that use whole numbers in contexts where they do not make sense, or they use unrealistic amounts that are too large or too small (break a small piece of candy into 8 parts, for example). We observe this as an important result that shows the lack of importance that many pre-service teachers ascribe to posing problems that make physical sense, which means that they don't establish connections between mathematics and the real world. According to Leavy and Hourigan (2022b), problems that draw on realistic contexts and consider student experiences successfully support and motivate the student, which is a fundamental and necessary indicator of their framework. Therefore, it is important to incorporate the analysis of the context and the context's reasonability when we pose or solve problems, making this characteristic relevant for the comprehension of the mathematical concepts implied.

The mathematical structures for both situations were *additive, multiplicative* and *additive-multiplicative*, as in Kiliç (2015). Some structures are rarely used, such as *concept, order* or a combination of both. It should be noted that the problems with a multiplicative structure are fundamentally associated with the use of the meaning of fraction as an operator (fraction times a natural number), and few problems were posed in which two fractions are multiplied or divided. This narrow contextual view of fraction is also reflected when analyzing the meanings, where *Part-Whole* and *Operator* predominate. It is true that the writing freedom in the PP-Num activity led them to pose some problems with meanings of *division, measure, ratio* or combinations thereof. At this point, we believe that the context offered in the second situation (PP-Context) conditions the type of problem that pre-service teachers are going to pose, directing it towards the *operator*.

Finally, the performance of the subjects in each of the situations presented was not similar. Practically all of the pre-service teachers posed two or three problems with all the necessary information when the situation was contextualized, and to a lesser extent when the situation was not contextualized and less constrained.

Problem posing is an unavoidable professional task in the daily work of teachers, one that must be part of the training of pre-service teachers. This study has allowed us to identify aspects to consider in potential training on problem posing involving fractions so that certain weaknesses related to the diversity of mathematical structures and meanings of fraction can be addressed. We were also able to confirm that the use of contextualized situations improves the performance of pre-service teachers.

Some results from this study should be taking into account for future training in problem posing for pre-service teachers. Although the situation selected, with or without context, does not substantially influence the plausibility of the problems, it is observed that the context helps to pose problems with a different structure than part-whole or order. Furthermore, it is observed that these situations let teachers to pose reasonable problems, which is necessary to get students to engage with the problems (Leavy & Hourigan, 2022b). This may suggest that training in problem formulation should begin with contextualised situations. Last but not least, it has been found in this study that teachers who are confronted with formulating various problems do so more successfully when a contextualised situation is provided than when it is not. The profiles of problem-posing teachers are more likely to be achieved with this type of task.

Problem posing is a task that requires more attention in the initial teacher education. We have found that the understanding of fractions is a limiting factor when pre-service teacher poses problems about fractions. But, in addition, the appropriate selection of the task-variable will help them to develop their ability to pose fraction problems with greater success and connecting these fractions with different structures and meanings.

CRediT authorship contribution statement

Alicia Bruno: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Resources, Supervision, Validation, Writing – original draft, Writing – review & editing. **Rut Almeida:** Conceptualization, Data curation. **Diana Sosa-Martín:** Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Supervision, Validation, Writing – original draft, Writing – review & editing, Resources. **Josefa Perdomo-Díaz:** Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Resources, Supervision, Validation, Writing – original draft, Writing – review & editing. **Israel García-Alonso:** Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Resources, Supervision, Validation, Writing – original draft, Writing – review & editing.

Declaration of Competing Interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Josefa Perdomo Diaz reports financial support was provided by Government of Canary Islands Ministry of Education and Universities.

Data availability

Data will be made available on request.

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