A new look at the Empirical Mode Decomposition

José L. Sanchez[a], Manuel D. Ortigueira[b], Raul T. Rato[c], Juan J. Trujillo[d]

[a] *Facultad de Informática, Universidad de La Laguna, 38271 La Laguna, Tenerife, Spain (e-mail:jose@isaatc.ull.es)*

[b] *UNINOVA and DEE of Faculdade de Ciências e Tecnologia da UNL[1]. Campus da FCT da UNL, Quinta da Torre, 2829 – 516 Caparica,Portugal (e-mail: mdo@fct.unl.pt)*

[c] *UNINOVA and Escola Superior de Tecnologia do Instituto Politecnico de Setubal, 2910-761 Setubal, Portugal (e-mail: raul.rato@estsetubal.ips.pt).*

[d] *Departamento de Análisis Matemático, Universidad de La Laguna 38271 La Laguna, Tenerife, Spain (e-mail: jtrujill@ullmat.es)*

Abstract

In this paper a look into EMD from the application point of view is done. Two important problems are faced and solutions proposed: a) the removal of false components and b) the increase in number of IMFs and computational time in long signals. To detect which IMFs are not presented in the original signal we propose an algorithm based on the spectral inversion and correlation. For decomposing very long signals a sequential sifting on segments of the signal in a sliding window is proposed.

Keywords: Empirical mode decomposition, Intrinsic mode function, long signal, spectral inversion

---

1. Introduction

The Empirical Mode Decomposition (EMD) as was proposed initially by Huang et al [4] is a signal decomposition algorithm based on a successive removal of elemental signals: the Intrinsic Mode Functions (IMF). These are continuous functions such that at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero. They are found through an iterative procedure called sifting that is a way of removing the dissymmetry between the upper and lower envelopes

[1] Also with INESC-ID, Lisbon.

in order to transform the original signal into an amplitude modulated (AM) signal. Moreover, as the instantaneous frequency can change from instant to instant, we can say that each IMF is a simmultaneously amplitude and frequency modulated signal (AM/FM). So, the EMD is nothing else than a decomposition into a set of AM/FM modulated signals [7, 8].

The original algorithm had some implicit difficulties [1, 7] that we tried to alleviate in a previously proposed algorithm where we used a parabolic interpolation to estimate the extrema and their locations. To render less severe the extremity effect we extrapolate the maxima and the minima. We inserted also a new stopping criterion in the sifting procedure by introducing two resolution factors (These assumptions led us to obtain an EMD implementation with better performances than previous algorithms[2][7].).

In the last years several modifications have been proposed to increase the performances of EMD, [2, 3, 5, 6, 9, 10, 11]. In some cases we wander if the introduced complexity compensates the quality increase. We think that in general we cannot say that they were really successful. So we are trying to maintain the beauty simplicity of the original algorithm and increase the reliability of the decomposition. On the other hand, we want to increase its applicability.

We must emphasize that EMD is merely a computational decomposition algorithm that expresses a given signal as a sum of simpler components. There is nothing telling us that the obtained components are really part of the signal at hand. We will make a brief study of this problem.

In practical applications there are several tradeoffs among: resolution, signal length, number of IMFs, and running time. In fact an increase in signal length leads generally to a corresponding increase in the number of IMFs and consequently in the running time that may become so high that the algorithm may become useless. The decrease in the resolution, decreases in general the number of IMFs and the running time but the quality of the decomposition also decreases.

The increase in the number of IMFs is a very important drawback because it may originate "false" components that are added to one IMF and subtracted to another one or appear isolated. So in general we have no guarantee to have IMFs that are really present in the original signal. This brings the need

[2] The routine can be found at http://www.mathworks.com/matlabcentral/fileexchange/21409-empirical-mode-decomposition

for an algorithm for detecting the extraneous components. Here we propose an algorithm to do it. It is based on the spectral inversion. To perform a spectral inversion, we assume that the signal is discrete-time and so bandlimited. The change of the sign of alternate samples produces a spectral inversion. With it we will devise a way of finding the real components.

In applications to long signals the number of components and the running time would be so high that the algorithm would be almost useless. To avoid it we propose here a reformulation of the procedure we proposed in [7] to deal with long signals. Essentially the algorithms consistis in cutting the original signal in overlapped segments and computing long IMFs one each time. This keeps the number of IMFs at an useful level as we will see later.

The paper outlines as follows. In section 2 we study EMD through some simple examples. The problem of removing non interesting IMFs is considered in section 3 and in section 4 we present a modified sifting to deal with very long signals.

Aquí conecta con el algoritmo largo

2. Some reflections about EMD

It is iportant to refer the usefulness of EMD in practical applications. The large number of papers published in the last years confirm the affirmation. Its usefulness is in the ability to decompose a given complicated signal into a finite set of narrowband signals without introducing any particular constraint on its characteristics. This makes easy the spectral estimation and creation of simple models.
Any way we must do some reflection about the questions posed by the algorithm.

- Meaning of the IMFs
  In general we are not able to establish any special connection between a given IMF and the structure (eventually tied with the underlying physics) of the original signal. This does not mean that we cannot do it in some particular situation as it is the case illustrated in figure 1 where we depict a tidal signal and its EMD.

  figure 1

A close look seems to point out that the most important IMFs are the two upper ones. The Fourier transform confirms such asumption since the peak frequencies of such IMFs correspond to the frequencies of the main components in the tidal signal: the positions of the Moon and the Sun relative to Earth and the Earth's rotation. The first has a period of about 12 hours and 25 minutes and the second has a period of 24 hours.

figure 2

These are clearly identified in the pictures. Even with a careful study it would be more difficult to give some meaning to some of the other components.

- Existence of false components in the IMFs
  The above example calls the attention to the existence of false components. This can be seen, for instance, in doing a comparison of strips 3 and 4 in figure 2 where we observe very similar spectra. This is a consequence of the numerical errors in sifting: one component is added in one IMF and subtracted in another one.

- The number of IMFs depends on the length of the signal
  In fact the number of components increases with the length of the signal. This is an unwanted feature of the algorithm that is connected with the false component generation. On the other hand this brings another drawback: the increase in the time required to do the decomposition.
  An example
  In a search for long range processes we made an experiment with the electric circuit of a heater fan. We acquired 2h of the signal with a sampling interval of 10ms. With it we computed the EMD of increasing length segments using the algorithm we described in [7] table 2 for 2 different resolutions [7]: $40, 45$.

  We made the computations on a current PC using MatLab. It is possible to decrease the computational time by implementing the algorithm with a high level languade like $C^{\#}$.

Table 1: IMFs and computational time for a heater fan signal

Resolution	Length	IMF's	Time
40	6,000	12	15
	30,000	14	70
	120,000	17	660
	240,000	18	1409
	360,000	18	1988
	480,000	20	3308
	600,000	20	4043
	720,000	21	5534
45	5,000	12	21
	10,000	16	156
	120,000	18	757
	240,000	20	9365 NO PUEDE SER
	360,000	19	3405
	480,000	21	6126
	600,000	21	7173
	720,000	23	12894

- The relation between the last IMF and the signal trend
  One application of EMD is the extraction of the trend assumed to be the last component. However this is valid only if the trend does not have any oscillation in the sense of having more than one extremum. In this case we could say that the last IMF was the trend, but the end effects distort it. We preffer to remove it because it must be quadratic. In this case it is simple to remove it without end effects by a least squares algorithm. In figure we illustrate this statement. We picked the last IMF of the EMD of the above referred tidal signal and retained the last IMF. We represent this IMF and the trend we got by a parabolic lewast squares adjustment. The end effects are clearly evidenced.

  figure 3

We can conclude that the main drawbacks of EMD are the false components and the large computational time when the signal is long. In the following we will propose solutions for these problems.

3. Which IMFs?

In this section we are going to address the issue of the false components. On being "empirical" the algorithm does not allow a pre-fixing number of IMFs. This implies the possibility of existing IMFs that are not "true" components of the signal and it is not easy to know which are really present in the original signal. In the following we will describe an algorithm for detecting which IMFs are really present in the signal. To do it, we will use the spectral inversion. This is very easy to obtain. For simplicity, assume that the signal at hand, $x(n)$, is discrete-time, with Fourier transform $X(e^{i\omega})$. Is is a simple task to show that the signal $(-1)^n x(n)$ has Fourier transform $X(e^{i(\pi-\omega)})$. We conclude that the spectral inversion is obtained by changing the sign of alternate samples.
With this we propose the following algorithm for detecting the IMFs really present in the signal.

1. Obtain the EMD of $x(n)$. Let X be the matrix of the M computed IMFs.
2. Put $y(n) = (-1)^n x(n)$ and obtain the corresponding N IMFs.
3. Produce the spectral inversion of all these N IMFs.
4. Correlate M IMFs obtained above with the N signals (not IMFs) got in 3. Those with correlations above a given threshold are considered components of the original signal.

figure 5

To illustrate the application of this procedure consider the decomposition of a signal that is a sum of two sinusoids with frequencies 4Hz and 23Hz and sampling frequency equal to 100Hz. The decomposition is shown in figure 4

figure 6

figure 7

As stated above we computed the correslations between all the signals in the two sets to obtain a correlation matrix C. Most values are below 10^{-3}. The main 3×3 submatrix was

0.9949	0.0022	-0.0003
0.0026	0.0042	0.0004
-0.0001	0.0029	0.0006

This matrix allows us to conclude that only 1 IMF is really meaningful.

4. Decomposing long signals

4.1. The problem

Let $x(t)$ be a given signal we want to decompose by EMD. As referred above the number of IMFs is not known in advance and normally increases with increasing the length of $x(t)$. This increments the computational burden, leading in some situations to very large computational times making the algorithm useless unless suitable actions are developped. One obvious procedure is to cut the signal into segments. However this can lead to poor results due to the following

- Different number of IMFs from segment to segment;
- The extremity effects introduce discontinuities at the junction points.

In the following we will introduce a solution for the problems having in mind to develop an algorithm suitable for decomposing very long signals. This situation is very common in mechanical, electrical, and bio-medical signal processing.

4.2. The solution

The idea behind the algorithm we are going to describe is yet a cut of the signal in segments. However, to avoid jumps in the spectral characteristics of the IMF, we allow the segments to overlap at the beginning and at the end. On the other hand, the procedure is applied to each IMF separately.
To understand the process, consider the signal at the top of 7. This is cut in 4 segments. Each segment has 600 points and the overlap is 25%: 150 points. To see how the reconstruction works we are going to do it. The first 450 points come from the 1^{st} segment; the next 150 points constitute the overlap zone and are a combination of the last 150 points of the first and the beginning of the second. The next 300 points come from the middle of segment 2, and so on. To avoid the end effects problem we apply a Tukey window to all the inner segments. To the first and last we apply at the end

in the first and at the beginning in the last.

figure 8

For a general formulation consider a signal of length L. Select the segment length N and the overlap M points.

1. The N^{th} length first segment, x_1, contributes directly with $N - M$ points.

2. The next M points come from the last M points in x_1 and the starting $M points$of x_2. At the overlapp regions we add simply the two signals since we have applied a window as referred above.

3. The next $N - 2M$ come from segment x_2.

The process continues till the end of the signal. This procedure is applied in the computation of the EMD of long signals. The idea is to do it for each IMF. Therefore,

1. Take the first segment of the signal and compute its first IMF. Count the number of used iterations; let it be N_i.

2. Take the second segment according to the pre-specified overlapping. Apply the window as prescibed above.

3. Compute the first IMF using N_i iterations. This must be done to have some guarantee that the amplitude of the IMF does not differ greatly from the amplitude of the first.

4. Combine the two IMF segments as we described above.

5. Continue with the process till the end of the signal; at this time and after combining the succesive IMF segments, we have a long first IMF.

6. Subtract the long IMF from the original signal and repeat the process.

In the IMF computation we adopted the algorithm that we described in [7, 8] for dealing with the extrema detection, as well as the stopping criteria. Concerning the extremity effects we proceed as in [7], since we used maxima

and minima outside the signal segment, but here we do not need to extrapolate them, since, excepting in the first and last segments, we have them. As the number of extrema decreases with the sifting iteration number, we can increment the window length in the last iterations.
In the following we illustrate the behaviour of the algorithm, mainly that it does not introduce artifacts due to the cut/past process. We used the EMD of an ECG signal obtained directly using the algorithm of [7] and new algorithm proposed here.

figure 9

figure 10

5. Conclusions

The Empirical Mode Decomposition is a technique to decompose any signal into a finite set of narrowband components, the Intrinsec Mode Functions, that do not necessarily give any insight into the underlying structure of the original signal. Besides the algorithm can supply "false" components in the sense that do not correspond to any spectral component of the signal. We proposed an algorithm based on the spectral inversion to identify those components and remove them from the decomposition.
On the other hand, the number of components and computational time increase dramatically when the length of the signal becomes large. A modified sifting algorithm to deal with long signals was proposed. Some comparison results were presented.

Acknowledgments

This work was partially funded by National Funds through the Foundation for Science and Technology of Portugal, under the project PEst-OE/EEI/UI0066/2011 and by project MTM2013-417 from Goverment of Spain.

References

[1] Chu, P.C., Fan, C. and Huang,N. *COMPACT EMPIRICAL MODE DECOMPOSITION: AN ALGORITHM TO REDUCE MODE MIX-*

ING, END EFFECT, AND DETREND UNCERTAINTY, Advances in Adaptive Data Analysis Vol. 4, No. 3 (2012) 1250017; DOI: 10.1142/S1793536912500173

[2] Feldman, M., *Analytical basics of the EMD: Two harmonics decomposition,* Mechanical Systems and Signal Processing, Volume 23, Issue 7, October 2009, Pages 20592071; DOI: 10.1016/j.ymssp.2009.04.002

[3] Guanlei, X., Xiaotong, W. and Xiaogang, X., *Improved EMD for the analysis of FM signals,* Mechanical Systems and Signal Processing, Volume 33, November 2012, Pages 181-196; DOI: 10.1016/j.ymssp.2012.07.003.

[4] Huang, N.E. ,Shen, Z., Long, S.R., Wu,M.L., Shih, H.H., Zheng, Q. , Yen, N.C., Tung,C.C., Liu,H.H. *The empirical mode decomposition and Hilbert spectrum for nonlinear and non-stationary time series analysis*, Proceedings of the Royal Society of London A 454 (1998) 903995; DOI: 10.1098/rspa.1998.0193.

[5] Jiang, H., Li,C., and Li, H. *An improved EEMD with multiwavelet packet for rotating machinery multi-fault diagnosis*, Mechanical Systems and Signal Processing, Volume 36, Issue 2, April 2013, Pages 225239;DOI: 10.1016/j.ymssp.2012.12.010

[6] Penga,Z.K., Tse,P. W., and Chua, F.L.,*An improved Hilbert-Huang transform and its application in vibration signal analysis,* Journal of Sound and Vibration, Volume 286, Issues 12, 23 August 2005, Pages 187205; DOI:10.1016/j.jsv.2004.10.005

[7] Rato, R. T., Ortigueira, M. D. and Batista, A. G., *On the HHT, its problems, and some solutions*, Mechanical Systems and Signal Processing, vol. 22, no. 6, pp. 1374-1394, August 2008;DOI: 10.1016/j.ymssp.2007.11.028.

[8] Rato, R. T., Ortigueira, M. D. and Batista, A. G., *The EMD and its use to identify system modes,* Proceedings of the International Workshop on New Trends in Science and Technology, Ankara, Turkey, November 3-4, 2008.

[9] Rehman, N. U.,Park, C., Huang, N. E., Mandic, D.P., *EMD VIA MEMD: MULTIVARIATE NOISE-AIDED COMPUTATION OF*

STANDARD EMD, Advances in Adaptive Data Analysis Vol. 5, No. 2 (2013) 1350007;DOI: 10.1142/S1793536913500076.

[10] Singh, P., Srivastava, P.K., Patney, R.K., Joshi, S.D., and Saha, K. *Non-polynomial spline based Empirical Mode Decomposition,* International Conference on Signal Processing and Communication (ICSC), 2013, 12-14 December 2013,Jaypee Institute of Information Technology, Noida, India; DOI:10.1109/ICSPCom.2013.6719829.

[11] Wang, L., Wang, Q., Wang, X., Zhang, M., and M, Y.,*An Improved Extension Method of EMD Based on SVRM,* Applied Mechanics and Materials Vols. 543-547 (2014) pp 2697-2701; DOI:10.4028/www.scientific.net/AMM.543-547.2697

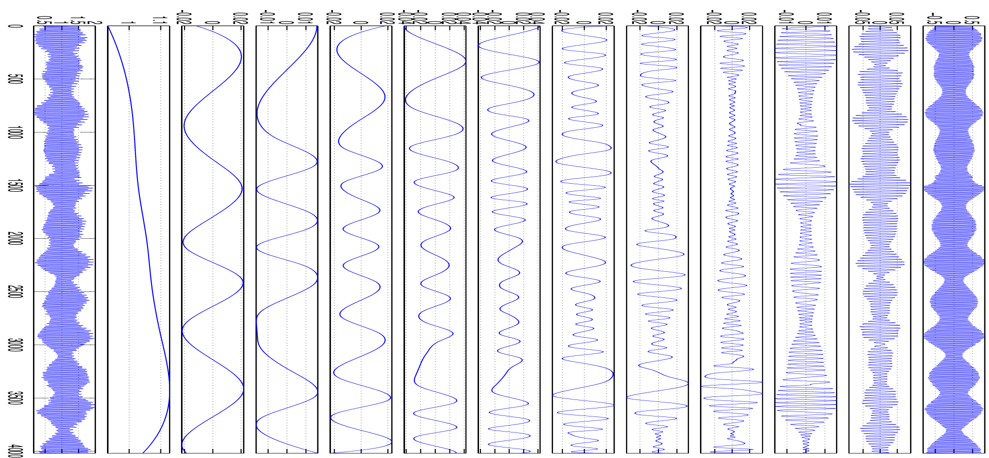

Figure 1: EMD of a tidal signal (in the last strip).

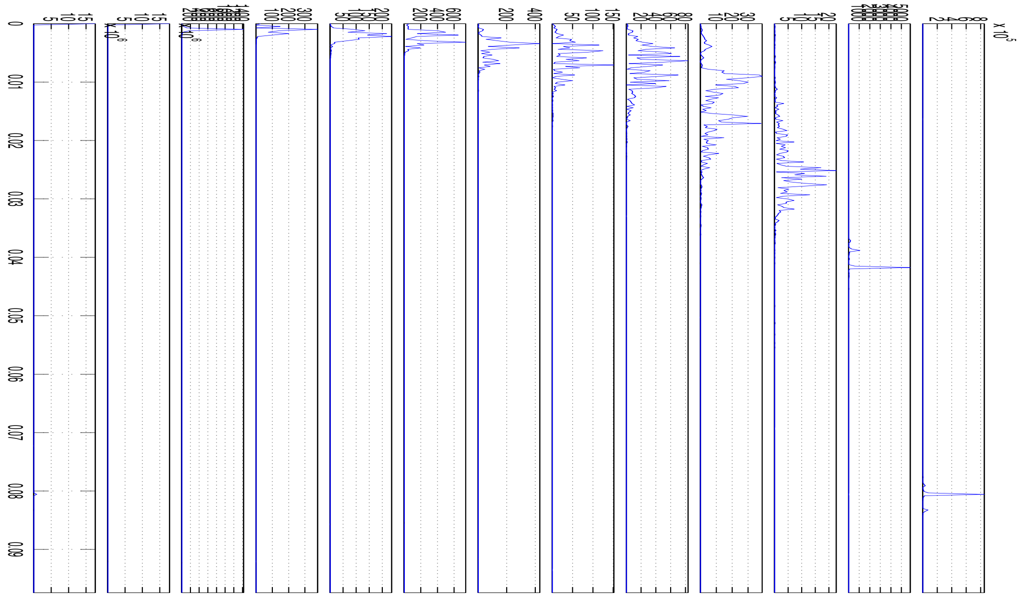

Figure 2: Absolute values of the Fourier transforms of the signals in 1.

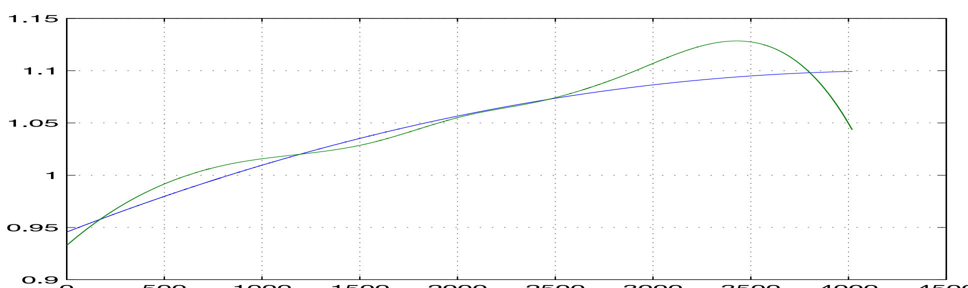

Figure 3: The trend of a tidal signal and the last IMF.

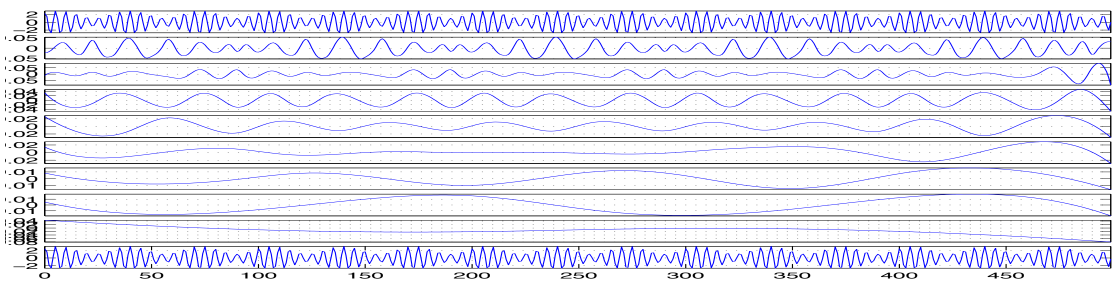

Figure 4: Sinusoidal signal with frequencies $4, 23$Hz (first strip) and its decomposition

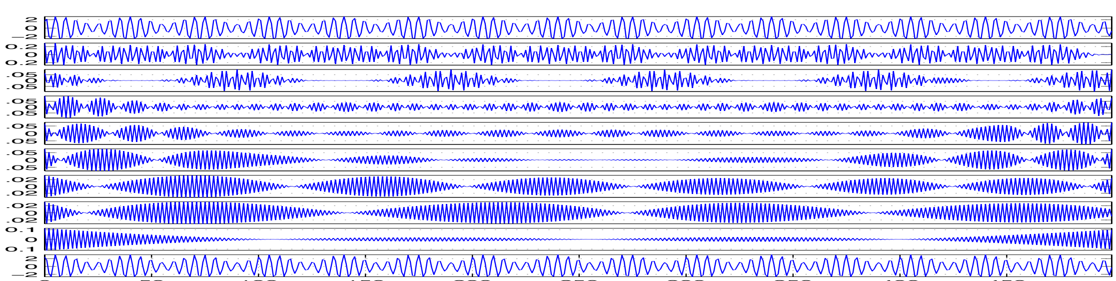

Figure 5: EMD of spectrally inverse of the sinusoidal signal with frequencies $4, 23$Hz

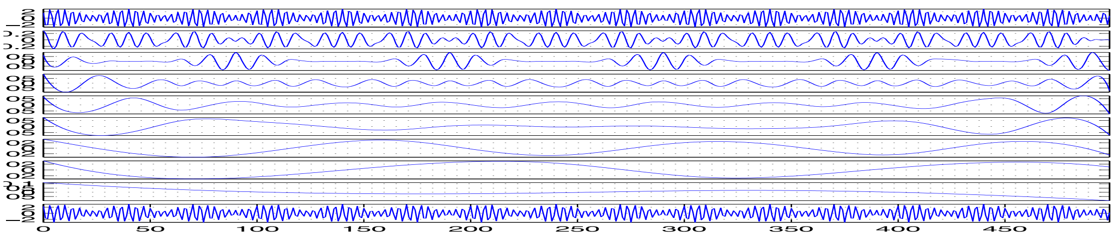

Figure 6: Spectrally inverse of the decomposition of the signal in 5

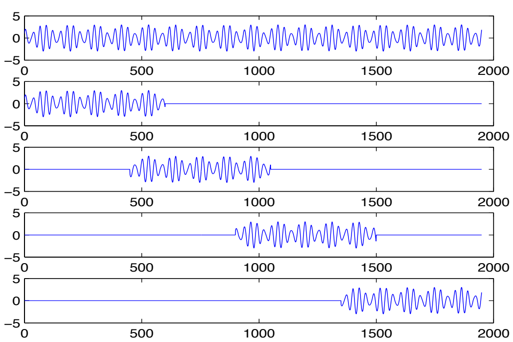

Figure 7: A signal and its corresponding segments.

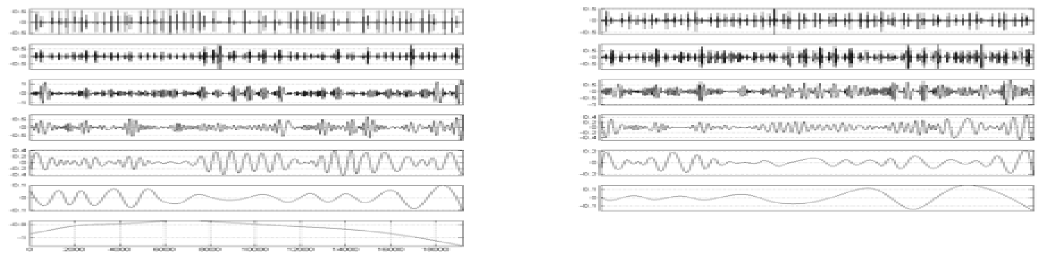

Figure 8: Decomposition of an ECG signal using the algorithm of [7]

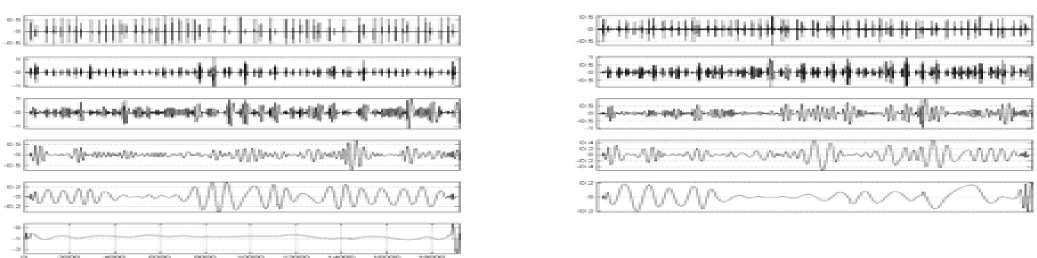

Figure 9: Decomposition of an ECG signal using the new algorithm