Trabajo de Fin de Grado

Universidad de La Laguna

Obtention of the Baryonic and Dark Matter power spectrum using certain approximations on the equations that rule the evolution of Baryonic Acoustic Oscillations

Sergio de Armas Rillo

Tutor: Dr. Juan Betancort Rijo

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1.1.1 Abstract

One of the most important considerations in the field of cosmology is the homogeneity and isotropy of our Universe. However, the existence of inhomogeneities is undeniable. To understand, then, how galaxies, clusters, superclusters, etc, appeared and formed, we have to study the evolution of the small primeval overdensities and underdensities present just after inflation. We will focus specifically on the evolution of those *proto-scales* (departures from the mean density, lets call them δ) in the specific time from before those scales enter the horizon until the time of decoupling. A purely gravitational approach is the one usually presented in textbooks, but here we will add two key elements to study: the radiation pressure, that affects baryonic matter until the time of decoupling, keeping this δ oscillating and efectivelly stalling its evolution; and the diffusion of photons, which will damp the oscillations and even erase small scales. We will study the three different evolutions that result of the introduction of these three elements in the case of a purely baryonic Universe. After this, we will introduce another ingredient, the Dark Matter, that will modify the behaviour of the scale factor a and contribute to the gravitational term of the equations, but will be always decoupled from radiation. The same study done before will be carried out again with this new element introduced. Finally we will obtain the power spectrum by probing different scales and obtaining the values of δ at the time of decoupling for each of these masses. This power spectrum contains all the relevant statistical information to characterize the distribution of structures in the Universe.

1.1.2 Resumen

Una de las consideraciones más importantes en el campo de la cosmología es la homogeneidad e isotropía de nuestro Universo. Sin embargo, es imposible negar la existencia de inhomogeneidades una vez nos vamos a escalas menores de $100Mpc$. Para entender cómo aparecen y se desarrollan las galaxias, cúmulos, supercúmulos, voids, etc nos vemos en la necesidad de estudiar la evolución de aquellas sobre y sub densidades presentes en los momentos inmediatamente posteriores al final de la inflación. Nos centraremos en la evolución de estas protoescalas (desviaciones sobre la densidad media del Universo) específicamente en el intervalo de tiempo desde que empezó a dominar la radiación (o más correctamente, desde que dichas escalas entran en el horizonte) hasta el momento del desacoplamiento de la radiación y la materia bariónica. Muchas veces nos encontramos con un tratamiento puramente gravitatorio de estas desviaciones (llamémoslas δ) al realizarse en períodos donde la radiación ha dejado de dominar y ser manejables analíticamente. Aquí sí que trataremos explícitamente los efectos producidos al añadir dos elementos cruciales a nuestro estudio: la presión de radiación, que afecta a la materia bariónica hasta que esta deja de estar acoplada a la radiación, lo que mantiene a la δ asociada a esta especie oscilando y que logra pausar su crecimiento de manera efectiva; y la difusión de fotones, fenómeno que amortiguará estas oscilaciones, llegando a borrar las δ asociadas a escalas suficientemente pequeñas (fenómeno conocido como Silk damping). Estudiaremos la diferentes evoluciones que conllevan la consideración de cada uno de estos factores en un universo puramente bariónico (esto es, un universo donde la única forma de materia sea la que conocemos, la bariónica). Lo que esperamos obtener es, en el primer caso en el que solo tenemos en cuenta la gravedad sin presión de radiación o difusión, un crecimiento estable con el

factor de escala (a) al cuadrado (dado que, sin la contribución de la materia oscura, DM de ahora en adelante, la radiación dominará hasta tiempos muy posteriores al desacople). En el segundo caso, la presión hará que nuestros δs vayan rápidamente hasta una situación oscilatoria estable alrededor de una posición de equilibrio ($\delta = 0$), oscilaciones de amplitud relativamente constante una vez estabilizadas. Estas oscilaciones se verán detenidas abruptamente en el momento en el que la expansión del universo impide que se produzcan más interacciones (scattering) entre fotones y electrones, fenómeno (ya mencionado) que se conoce como desacople y ocurre en $t = t_{dec}$. A partir de este momento, la materia y la radiación ya no forman un fluido conjunto y evolucionan de manera independiente. La introducción de la difusión de fotones tendrá como resultado el amortiguamiento de las oscilaciones inducidas por la presión de radiación, proceso que trataremos de manera muy aproximada pero que intentaremos justificar. Para escalas pequeñas el desacople llega demasiado tarde, y el efecto de amortiguación será tan importante que δ será borrado. Este proceso es conocido como Silk damping y aquellas escalas de masa menor que la de la llamada masa de Silk no sobrevivirán.

Tras analizar la evolución de δ_B, δ_r en un universo puramente bariónico, el siguiente paso consiste en añadir a la receta la materia oscura. La primera consecuencia de esta acción es la modificación del comportamiento del factor de escala $a_{(t)}$. Por un lado cambia el valor del parámetro asociado a la materia en la ecuación de Friedmann que usamos para describir $\dot{a}$: tenemos un nuevo y mayor valor de Ω_m, siendo Ω_m el parámetro de densidad de la materia, resultado de dividir la densidad de materia entre la densidad crítica del Universo en el presente. Por otro lado, este cambio no debería tener un gran impacto para las épocas dominadas por radiación. Pero, en un universo con $\Omega_m = 0{,}3$ en lugar del 0,04 que aporta la materia bariónica, el momento en el que la influencia en la expansión del universo de la materia y la radiación se igualan ocurre mucho antes, antes que el desacoplamiento. Por tanto, nuestro factor de escala se comportará de manera diferente en este nuevo cosmos. La segunda consecuencia, más importante, es que tenemos ahora un segundo fluido que añadir al fluido radiación-bariones, el de la DM. Este componente no interactuará con el resto sino es gravitatoriamente, pero influenciará su evolución a partir del momento en que el Universo deja de estar dominado por la radiación, $t = t_{eq}$. Continúa creciendo mientras los bariones y la radiación oscilan alrededor de $\delta = 0$. Bajo determinadas circunstancias contribuirá a reactivar el crecimiento de las inhomogeneidades bariónicas, pero el estudio de la evolución de δ_{DM} tiene más interés para saber cómo afecta la materia oscura a los bariones una vez estos se han desacoplado de la radiación: respecto a una evolución puramente gravitatoria, el crecimiento de las escalas de DM también se ha estancado, pero si lo comparamos con el sufrido por los bariones, se puede considerar que las escalas de materia oscura han continuado creciendo. En cuanto los bariones no estén ligados a la radiación la principal influencia en su desarrollo será la de la materia oscura, y el valor de δ_{DM} que se tiene en el momento del desacoplamiento. Para intentar obtener los comportamientos descritos en este resumen resolveremos una serie de ecuaciones. Obtener dichas ecuaciones no es parte del trabajo; todas ellas han sido dadas por el tutor de este trabajo, el doctor Juan Betancort Rijo, en comunicación personal. Lo que se ha hecho es comprobar si los comportamientos de las soluciones de estas ecuaciones se ajustan a lo esperado y justificar su validez, cuanto menos cualitativa, y por tanto la idoneidad de las mismas con objeto de ser usadas como herramienta pedagógica o incluso como test para posibles propuestas de valores para los ingredientes del universo, dependiendo de la fidelidad que guarden los resultados obtenidos con el conocimiento que tenemos de estos fenómenos.

1.2 Theoretical considerations

En esta sección presentaremos las bases del tratamiento que se suele hacer del comienzo de las desviaciones de la densidad media del Universo que dieron lugar a las estructuras que vemos hoy en día. Empezaremos decribiendo brevemente cómo se trata al cosmos como un todo homogéneo y la principal solución a las ecuaciones de Einstein que deriva de la consideración del principio cosmológico. Acto seguido, introduciremos un primer tratamiento para la obtención de pequeñas fluctuaciones lineales.

1.2.1 An homogeneous Universe

The field of cosmology has always been one of utmost difficulty for a number of factors. First and foremost, we don't have an easy way of accessing the necessary data to test hypothesis and derive properties from. Even today, experimental projects proposed from this field are expensive and cannot be reproduced at a small scale. But, as everything that heavily depends on technological proficiency, this thick veil that covers the coveted empiric data has been being thinned year by year. However, there is another layer of complication when dealing with the study of the cosmos, and it is that the cosmos is, inherently, extremely vast. Years after the publishing of Einstein's article in which his Field Equations were introduced we still were not sure whether or not other galaxies existed or if our star counts were in fact describing the entirety of the Universe (we can remember the famous Great Debate held on 1920). After the acceptance of nebulae as other galaxies outside our own, there were continuous efforts to count and place those structures. These observations, mainly due to the work of Hubble (1926-1934) and Shapley, pointed to a somewhat uniform distribution of galaxies, but with some reservations. Amid this new debate lied the famous Einstein Field Equations (EFE for now on)

$$R^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}R = \frac{8\pi G}{c^4}T^{\alpha\beta} \tag{1}$$

the angular stone upon which all orthodox cosmological research should be sustained, and the one with more interest for both theoretical investigators and contemporaneous students looking for a fundamental comprehension of our world. While, as has been said, the search of an order for the structures of the Universe was taking place, an exact, non-trivial solution for these equations was found, one that would eventually prove itself to be the best approach to EFE from a pedagogical point of view. Uncountable physic students from around the world have taken their first steps in cosmology held by the helpful hand of the Robertson-Walker metric and the Friedmann model of the Universe. We can define the scale factor of the Universe as a, with

$$x_{(t)} = a_{(t)}r \tag{2}$$

and r the comoving coordinates of a point in the space. This factor describes the expansion of the Universe. The metric mentioned before, in its general form, would be:

$$ds^2 = -dt^2 + a_{(t)}^2(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) \tag{3}$$

And from it we can obtain the following Friedmann equations:

$$\frac{\dot{a}^2 + kc^2}{a^2} = \frac{8\pi G\rho + \Lambda c^2}{3} \tag{4}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3p}{c^2}) + \frac{\Lambda c^2}{3}$$

The reader is probably familiar with these equations, or at least with some different way of presenting them. Every undergrad student has worked with them, and this model was used extensively for 30 years. The reason? For us students it is a clear one: it drastically simplifies our work. And these equations are based in two fundamental assumptions, summed up in the cosmological principle: that, in large scales, the Universe is homogeneous and isotropic. We won't dwell on the validity of this statement. Let us say that it works perfectly as a very accurate first approximation to our Universe. But there is an evident truth: at some scale this principle is not correct. To beging with, we are no more that punctual overdensities in the Universe, albeit small ones. But there certainly are larger structures out there. Galaxies, yes, but even them are small. Clusters, superclusters, voids, etc, are the object of study of the large scale structure of the Universe, a field that has been growing since we have been able to extract data that departs from the first approximations that Friedmann's equations are. Departing from the homogeneity is not a trivial issue, but there is a silver lining: as long as the deviations from the mean are small, we can treat them as first-order perturbations and, more importantly, we can make use of a newtonian approximation within the horizon.

1.2.2 Small perturbations

Picture the Universe at its earliest stages. If we take the Big Bang as happening at $t = 0$, we will have a Universe affected by the complexities of quantum mechanics until around $t = 0.7t_{Planck}$ (see chapter 5 in [3] for a justification of this rough estimate). This implies the existence of quantum fluctuations in regions of the cosmos that, after inflation, would lose causal contact. We are interested in how these fluctuations evolve throught time, from the moment the inflation stopped until they can be observed. Specifically, we will focus on a specific time interval from the start of the radiation-dominated Universe (inmediately after inflation) until the time of decoupling. What happens during this time has observable repercussions in the Universe, but it is difficult to find texts that focus too much on this era, some of them even glossing over the evolution of inhomogeneities during the radiation-dominated epoch and only studying the growth of structure when matter is already the main component of the Universe. This is due to the difficulty of an analytical treatment of this radiation-dominated epoch, so the results are presented almost qualitatively.

In general, the equations that rule these cosmological fluctuations are extremely complicated. Few physicist try to approach them numerically, and even less would dare to treat them analitically. Most cosmologist have resolved to computational simulations (perhaps the most famous would be CMBFAST) and it is extremely difficult to truly understand the physics that hide behind such computational behemots. However, there are certain approximations, not valid for the entire history of those fluctuations but appropiate for some regimes, that could open these equations to a simple numerical treatment. The results are not expected to be extremely accurate, but will provide some insight in the behaviour of the components of the universe during the obscure times that preceded the time of decoupling.

The main consideration that we will take (besides the small size of the perturbations) is the aforementioned Newtonian approximation. We can can follow chapter II in [2] for justifying the validity of this approximation.We cannot assure that we will work in a completely flat space, at least when we compute the values of $a_{(t)}$, but we can always find coordinates that will make the metric tensor and its first derivatives take the form $g_{ij} = \eta_{i,j}, g_{ij,k} = 0$, that is, the Minkowski form, at least locally. Remeber, the metric tensor determines the proper spacetime distance between two events: $ds^2 = g_{ij}dx^i dx^j$. We can change our coordinates ($y^i = y^i_{(x^i)}, dy^i = \frac{\partial y^i}{\partial x^j}dx^j$) to make the observer, who we assume is freely falling following a geodesic, to stay at $y^\alpha = 0$. We can describe the acceleration suffered by a particle at a distance r from the world line along which an observer is moving by perturbing this metric ($g_{ij} = \eta_{ij} + h_{ij}$), h_{ij} being small and proportional to r^2. We then can apply the standard weak field approximation and find (Landau and Lifshitz, 1979):

$$R_{00} = -\frac{1}{2}(h_{ij,00} - h_{i0,j0} - h_{j0,i0} + h_{00,ij}) = \nabla_r^2\Phi \tag{5}$$

$$g_{00} \equiv c^2 + 2\Phi \tag{6}$$

An ideal fluid of pressure p, density ρ and with a velocity much smaller than the velocity of light, the zero-zero comoponent of the EFE is then

$$\nabla_r^2\Phi = 4\pi G(\rho + 3\frac{p}{c^2}) - \Lambda \tag{7}$$

(we will set Λ to 0), and we will have a geodesic equation

$$\frac{d^2r^\alpha}{dt^2} = -\Phi_{,\alpha} \tag{8}$$

The previous two equations are pure Newtonian equations (if there is no Λ nor strong radiation background). But we were trying to see where these considerations are valid. If both the observer and the observee have $v \ll c$, and $\Phi \ll c^2$, Λ is almost 0 and $\Omega = 1$, then

$$R \ll cH^{-1}$$

For $H_0 \simeq 1.023 \cdot 10^{-10}y$, this means $R \ll 10^{28}cm \sim 10^3 Mpc$. For the lower limit we can take supermassive black holes with a Schwarzschild radious of $10^{14}cm$. Then, this Newtonian approximation would work in scales $10^{-8}Mpc \ll r \ll 10^3 Mpc$. Virgo's supercluster has a mass of $\sim 10^{15}M_\odot$, that corresponds to a $r \sim 35Mpc$. Therefore, and as we won't treat scales significatively larger than this supercluster, and we won't deal with relativistic matter, the newtonian approximation is a good one.

There is another consideration to make. We will consider all the species to study to behave as ideal fluids. In one case we will have a radiation-matter fluid, in the other situation considered we will add to this fluid another, independent one, formed of cold dark matter (CDM). This hydrodynamic limit is justified if we have not reached t_{dec}. Since the end of inflation and until this time, the photon-electron scattering rate is larger than the expansion rate of the Universe. This implies that baryonic matter and radiation are tighly coupled, and radiation is in local thermal eqilibrium with the baryonic fluid (8.1 in [1]). We start with the equations for the baryonic ideal fluid , being $x \equiv$proper distance and $u \equiv$proper velocity:

$$\begin{aligned} \frac{\partial\rho}{\partial t} + \nabla_x \cdot (\rho u) &= 0 \\ \frac{\partial u}{\partial t} + (u \cdot \nabla)u &= -\nabla\Psi_{tot} - \frac{1}{\rho}\nabla p \\ \nabla^2\Psi_{tot} &= 4\pi G\rho_{tot} \end{aligned} \tag{9}$$

Here we have ρ_{tot} as the sum of the relevant gravitational elements: in this case, ρ_B and ρ_r. If we had DM, it would appear here too. The term $\frac{1}{\rho}\bigtriangledown p$ comes from the coupling of radiation with baryons and describes radiation pressure. Radiation pressure appears much earlier than the gas pressure the baryons would have were they alone. We will see how this term affects the evolution of the fluid.

So far we haven't introduced true perturbations. At this point, we can define the density contrast of baryons. If $\bar{\rho}_b$ is the mean density of baryons, we have that

$$\rho_{(x,t)} = \bar{\rho}_b[1 + \delta_{b,(x,t)}] \tag{10}$$

with δ_b being this dimensionless density contrast. We can apply the same definition to find δ_r and δ_{DM}. We are going to apply a perturbation to the ecuations fo the baryon fluid, using δ_b and the perturbed potential and velocity ϕ, v:

$$\phi = \Psi - \phi_{FRW}, v = u - Hx \tag{11}$$

$$\delta_b = \frac{\rho - \bar{\rho}_b}{\bar{\rho}_b}, \delta p = c_s^2 \delta\rho \tag{12}$$

ϕ_{FRW} is the potential for background expansion ($\phi_{FRW} = -\frac{\ddot{a}}{2a}x^2$).

We change from proper to comoving coordinates ($r = \frac{x}{a_{(t)}}$). Using

$$(\frac{\partial}{\partial t})_x = (\frac{\partial}{\partial t})_r - Hr\frac{\partial}{\partial r}; (\frac{\partial}{\partial x})_t = \frac{1}{a}(\frac{\partial}{\partial r})_t; H = \frac{\dot{a}}{a} \tag{13}$$

and the velocity $v/u = \dot{a}r + v$. With these relations we can convert the previous equations to the new comoving coordinates (r, t). We now linearize (7) and get

$$\ddot{\delta}_{B,k} + 2\frac{\dot{a}}{a}\dot{\delta}_{B,k} = 4\pi G(\bar{\rho}_\alpha \delta_\alpha) - \frac{c_s^2 k^2}{a^2}\delta_{B,k} \tag{14}$$

$\bar{\rho}_\alpha \delta_\alpha$ is the sum of this product for $\alpha = B, r, DM$; c_s is the speed of the propagation; and k is the wavenumber associated to a certain scale. The reader can find a more detailed calculation in [4]. Since matter and radiation are tighly coupled, we can establish a relation between δ_B and δ_r for the adiabatic mode (the one we are interested in) (Zel'dovich, 1965):

$$\delta_r = \frac{4}{3}\delta_B$$

It is convenient to perform a fourier transform on δ:

$$\delta_B \to \delta_{B,k}$$

If we start from the equations just obtained, assuming a Universe with only radiation and baryonic matter, we could reach the following expression:

$$\ddot{\delta}_{B,k} + 2\frac{\dot{a}}{a}\ddot{\delta}_{B,k} = 4\pi G(\bar{\rho}_B \delta_{B,k} + \bar{\rho}_r \delta_{r,k}) - \frac{1}{3}\frac{\bar{\rho}_r}{\bar{\rho}_B + \bar{\rho}_r}\frac{c^2}{a^2}k^2 \delta_{r,k} \tag{15}$$

We will not detail the obtention of said expression, as it is not the aim of this text, but we will check if it produces the expected solutions. We will use the relation between $\dot{\delta}_{r,k}$ and $\dot{\delta}_{B,k}$. We can add a new term to this expression if we take into account photons diffusion due to the changes on the local entropy o the Universe caused by the oscillations of inhomogeneities of baryons and radiation. This diffusive analysis is extremely complicated, but if we assume the photon flux to follow a Fourier type law, an approximate expression can be found (personal communication, [6]):

$$\dot{\delta}_{r,k} = \frac{4}{3}\dot{\delta}_{B,k}(\frac{2}{\sqrt{1+4(1.493 \cdot 10^6[Mpc]ka^2)^2}})^{0.9} - \frac{\mu k^2}{a^2}\delta_{r,k} \tag{16}$$

which will modify the solution obtained for $\delta_{B,k}$. We have also defined μ as

$$\mu = \frac{1}{3}\frac{1}{n_{e(t)}\sigma_T}c \tag{17}$$

n_e is the number density of electrons, and σ_T the Thomson cross-section. We have assumed conservation of photons and considered the photon flux intensity $\vec{J}_\gamma$ as

$$\vec{J}_\gamma = -\mu\vec{\nabla}n_\gamma \tag{18}$$

An important aspect to compute the power spectrum $P_{(k)}$ is that each mode δ_k is an independent variable. This means that:

$$\langle\delta_{k(t_i)}\delta^*_{p(t_i)}\rangle = (2\pi)^3 P_{(k,t_i)}\delta_{Dirac(k-p)} \tag{19}$$

We have to note that we have assumed a constant ionization fraction for the obtention of the equations. This consideration may be problematic for times near decoupling, when the ionization fraction changes rapidly during a brief time interval. One last consideration that we will take in order to compute the power spectrum $P_{(k)}$ is the dimensionless relation

$$\Delta^2_{(k)} = P_{(k)}, \Delta^2_{(k)} = \delta^2_{(k)}\frac{2\pi^2}{k^3} \tag{20}$$

1.3 Objectives

Presentamos las ecuaciones a resolver y los resultados que esperamos encontrar. In the previous section we ended up with a series of equations to solve.

1.-Purely baryonic Universe without radiation pressure:

$$\begin{aligned} \ddot{\delta}_{B,k} + 2\frac{\dot{a}}{a}\ddot{\delta}_{B,k} &= 4\pi G(\bar{\rho}_B\delta_{B,k} + \bar{\rho}_r\delta_{r,k}) \\ \dot{\delta}_{r,k} &= \frac{4}{3}\dot{\delta}_{B,k} \end{aligned} \tag{21}$$

2.-Purely baryonic Universe with radiation pressure:

$$\begin{aligned} \ddot{\delta}_{B,k} + 2\frac{\dot{a}}{a}\ddot{\delta}_{B,k} &= 4\pi G(\bar{\rho}_B\delta_{B,k} + \bar{\rho}_r\delta_{r,k}) - \frac{1}{3}\frac{\bar{\rho}_r}{\bar{\rho}_B + \bar{\rho}_r}\frac{c^2}{a^2}k^2\delta_{r,k} \\ \dot{\delta}_{r,k} &= \frac{4}{3}\dot{\delta}_{B,k} \end{aligned} \tag{22}$$

3.-Purely baryonic Universe with radiation pressure and photon diffusion:

$$\begin{aligned}\ddot{\delta}_{B,k}+2\frac{\dot{a}}{a}\ddot{\delta}_{B,k} &= 4\pi G(\bar{\rho}_B\delta_{B,k}+\bar{\rho}_r\delta_{r,k})-\frac{1}{3}\frac{\bar{\rho}_r}{\bar{\rho}_B+\bar{\rho}_r}\frac{c^2}{a^2}k^2\delta_{r,k} \\ \dot{\delta}_{r,k} &= \frac{4}{3}\dot{\delta}_{B,k}\left(\frac{2}{\sqrt{1+4(1.493\cdot 10^6 ka^2)^2}}\right)^{0.9}-\frac{\mu k^2}{a^2}\delta_{r,k}\end{aligned} \quad (23)$$

If we add dark matter (DM from now on), we will have the same set of equations for $\delta_{B,k}$ adding to the gravitational term $(4\pi G(\cdots))$ the DM inhomogeneity $\bar{\rho}_{DM,k}\delta_{DM,k}$, but we will also have the equations that govern the evolution of these $\delta_{DM,k}$:

$$\ddot{\delta}_{DM,k}+2\frac{\dot{a}}{a}\dot{\delta}_{DM,k}=4\pi G(\bar{\rho}_B\delta_{B,k}+\bar{\rho}_{DM,k}\delta_{DM,k}+\bar{\rho}_r\delta_{r,k}) \quad (24)$$

with $\delta_{B,k}, \delta_{r,k}$ behaving differently in each of the previously mentioned cases (only gravity, with radiation pressure and with photon diffusion).

We will try and solve these equations keeping in mind that some crude approximations have been made. For starters, we don't take into account Λ (althought it is highly justified, as its dominance won't start until much later than the time of decoupling); we consider the hydrodinamical limit for radiation; we assume a fixed ionization fraction and photon-to-electron ratio (one of the most important approximations for the treatment of photon diffusion); we treat the Universe as newtonian, as justified before; we only treat the limit of small fluctuations... However, we expect the results to be at least good indicatives of the behaviour of the irregularities in the timeframe studied. We hope to be able to visualize these fluctuations in a manner rarely seen in most texts, as this part of the evolution of irregularities is commonly dismissed in order to jump straight ahead into the realm of matter-domination (even when the study of the highest amplitude for the fluctuations of the baryon-photon fluid can provide a useful insight into the baryon density of the Universe).

We will then solve the aforementioned equations and obtain the evolution of $\delta_{B,k}$, $\delta_{r,k}$ and $\delta_{DK,k}$ when applicable. We will justify then the common statements broad-stroked by many lecturers on large-scale courses regarding this issue, and, if the results are in accordance with what we know already on the subjet, we will have a light, easy-to-use test for new proposals of Universes. Finally, we will obtain the power spectrum of baryons and DM in different epochs (the observable quantity more readily accesible).

2 Solving the equations

En esta parte procederemos a detallar y evaluar los resultados de la resolución numérica de las ecuaciones presentadas previamente. Comenzaremos representando la evolución de los parámetros δ de bariones y DM en cada uno de los universos estudiados (puramente bariónico sin presión, con presión, con difusión de fotones, etc), detallando algunos aspectos críticos de las mismas, como el momento en el que comienzan las oscilaciones bariónicas. Representaremos el espectro de potencias de la materia oscura y la bariónica, que contiene la mayor parte de la información sobre la distribución de estructura a gran escala del universo.
Finalmente, haremos un último análisis de los resultados obtenidos, su validez, utilidad y posible cursos de acción a tomar partiendo de ellos.

2.1 Methodology

Cosmology is a delicate subject. It does not matter how simple or complicated the expressions we are going to work with are, they are very sensitive to changes in the definition of the different constants (sometimes not so constant here). We have a great number of inter-relations between formulae, and one small change can propagate with nefarious consequences for the rest of the work carried out. This is why it is convenient to tread carefully and methodically when translating our paper-based equations to the realm of actual numerical values in our program. Let us begin writing again the equations to solve in the first part of our work:

$$\begin{aligned}\ddot{\delta}_{B,k} + 2\frac{\dot{a}}{a}\ddot{\delta}_{B,k} &= 4\pi G(\bar{\rho}_B\delta_{B,k} + \bar{\rho}_r\delta_{r,k}) - \frac{1}{3}\frac{\bar{\rho}_r}{\bar{\rho}_B + \bar{\rho}_r}\frac{c^2}{a^2}k^2\delta_{r,k} \\ \dot{\delta}_{r,k} &= \frac{4}{3}\dot{\delta}_{B,k}\left(\frac{2}{\sqrt{1+4(1.493\cdot 10^6 ka^2)^2}}\right)^{0.9} - \frac{\mu k^2}{a^2}\delta_{r,k}\end{aligned} \quad (25)$$

We are probably familiar with many of the elements of these equations, their physics not being very complicated, as the scale factor, the wavenumber k, the speed of light, the densities (that, from now on, will be writen as ρ_α instead of $\bar{\rho}_\alpha$)... But, if we are to work with them, we can find that the actual values of these quantities are very fickle. We can start with the main variable in most relations in cosmology: the scale factor a. At undergrad courses, it is the norm to use the scale factor of certain epochs to ease its obtention. Therefore, we would work with an $a \propto t^{1/2}$ in the radiation-dominated epoch, $a \propto t^{2/3}$ in the matter-dominated one, etc. However, it is convenient to find a proper value for a, as it is not very tasking and extremely rewarding. We will use

$$\frac{\dot{a}}{a} = H_0\sqrt{\Omega_{0,m}a^{-3} + \Omega_{0,rad}a^{-4} + \Omega_\Lambda + \Omega_{0,K}a^{-2}} \quad (26)$$

For a purely baryonic Universe, $\Omega_{0,m} = \Omega_{0,b}$, with a value set as $\Omega_{0,b} = 0.04$ (in corcondance with the results of WMAP); $\Omega_{0,rad} = 1.59\cdot 10^{-4}$ (we have taken into account the contribution of three species of neutrinos, not only the contribution of photons), and Ω_Λ will be 0. The values of these Ω are not very important: one of the desired results of this work is to be able to modify these kind of quantities and study the results of such change. Since we want $(1-\Omega) = 0$, $(\Omega = \sum_\alpha \Omega_\alpha)$, we introduce a curvature density, $\Omega_{0,K} = 1 - \Omega_{0,m} - \Omega_{0,rad}$, that will eventually be irrelevant, as in the stages studied its contribution will be absolutely dwarfed by both radiation and matter, even if $\Omega_{0,K}$ is significatively larger than the other density parameters.

We have to solve the differential equation

$$\dot{a} = aH_0\sqrt{0.04a^{-3} + 1.59\cdot 10^{-4}a^{-4} + 0.9598a^{-2}}$$

where we have taken H_0 to be $H_0 = 1.023\cdot 10^{-10}y^{-1}$. We will finish our integration at the time of decoupling, approximating this time to the time of recombination. In many texts it is taken as happening at $z = 1300$, $a = \frac{1}{1+z}$; until this time, we asume the **comoving** electron density to be constant. As choosing a bad value of a_i could propagate to the rest of equations, and because this method is easier to implement by being able to control the timestep for the values of a obtained, first we are going to integrate since a very early time with a very small scale factor (for example, $t = 3s \Rightarrow a \sim 3\cdot 10^{-10}$) until a later time that is still small enough compared to the time at which the scales we are going to study are deep outside the horizon (1 *year*), where we start the integration again with the obtained initial value of a until the time of decoupling. What we get is

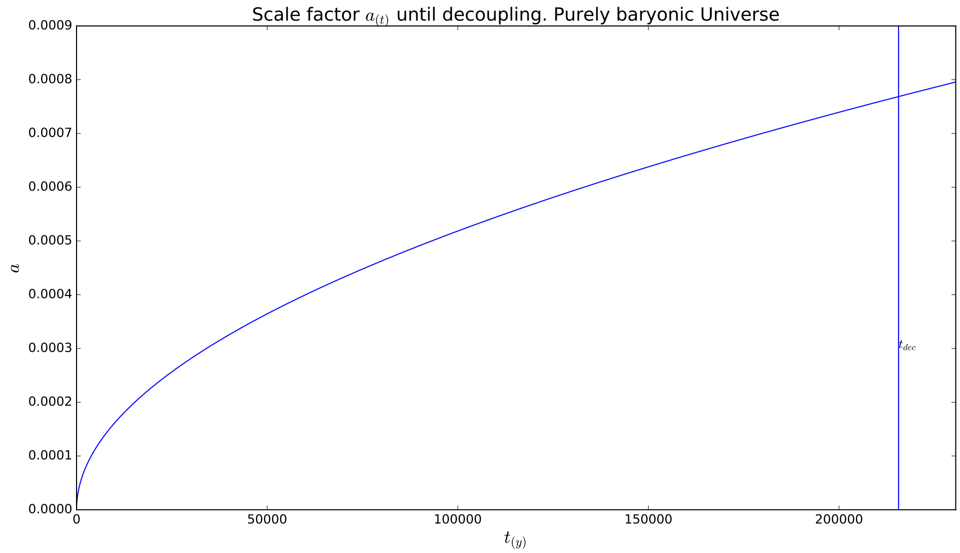

Figure 1: *We will take the solutions of our equations for the fluctuations up until we reach* a_{dec}

With $t_{dec} = 215607y$. We can compare this result with the one we would get if we had a radiation-only Universe. In this case, $a = \sqrt{2H_0\sqrt{\Omega_{0,rad}t}}t$, so we can plot this quantity and the previous result for a against $\sqrt{t}$

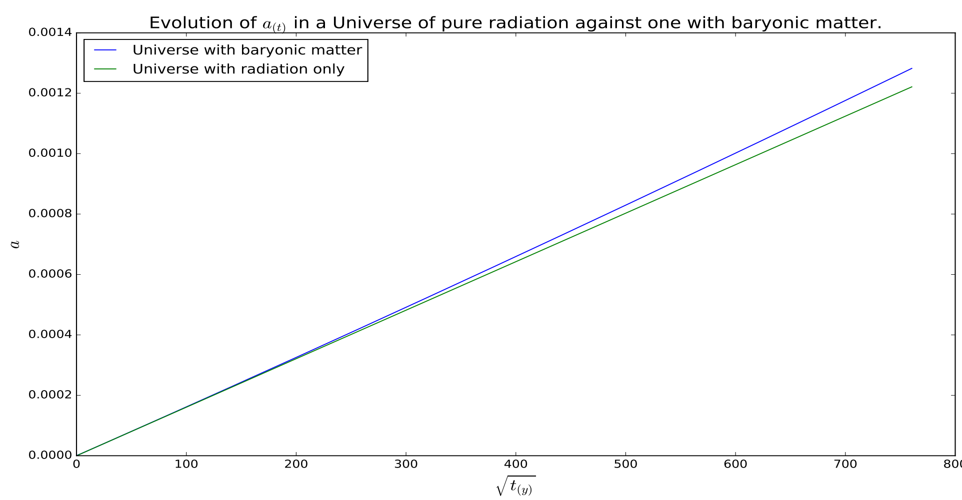

Figure 2: *We do not depart from the evolution of a radiation-only Universe until times near*t_{dec}

We now have a set of values for $a_{(t)}$ from $t = 1y$ to $t = t_{dec}$. From equation (26) we can also obtain $\dot{a}_{(t)}$. Once we have a satisfactory value for the scale factor at all important t, now we must make a little bit of work on the equations to make them manageable. Let us present one of the most important equations in cosmology, the Friedmann equation, in one of its many forms

$$\frac{3}{2}(\frac{\dot{a}}{a})^2 = 4\pi G(\rho) \tag{27}$$

with $\rho = \rho_B + \rho_r$ and we have disregarded the curvature term. We can see that, if we multiply and divide the gravitational term in equation (25) by $(\rho_B + \rho_r)$, and using the previous equation, we have

$$\ddot{\delta}_{B,k} + 2\frac{\dot{a}}{a}\ddot{\delta}_{B,k} = \frac{3}{2}(\frac{\dot{a}}{a})^2(\frac{\rho_B}{\rho_B + \rho_r}\delta_{B,k} + \frac{\rho_r}{\rho_B + \rho_r}\delta_{r,k}) - \frac{1}{3}\frac{\rho_r}{\rho_B + \rho_r}\frac{c^2}{a^2}k^2\delta_{r,k} \tag{28}$$

We still have the same problematic ρ terms . To get rid of them we can use the known Ωs:

$$\frac{\rho_B}{\rho_B + \rho_r} = \frac{1}{1 + (\rho_r/\rho_B)}, \frac{\rho_r}{\rho_B} = \frac{\Omega_{0,r}}{\Omega_{0,B}}\frac{1}{a} \Rightarrow \frac{\rho_B}{\rho_B + \rho_r} = \frac{1}{1 + \frac{1}{a}(\Omega_{0,r}/\Omega_{0,B})} \tag{29}$$

In the same fashion

$$\frac{\rho_r}{\rho_B + \rho_r} = \frac{1}{a(\Omega_{0,B}/\Omega_{0,r}) + 1}$$

The ratios $\frac{1}{1+\frac{1}{a}(\Omega_{0,r}/\Omega_{0,B})}$ and $\frac{1}{a(\Omega_{0,B}/\Omega_{0,r})+1}$ will be named Ω_B and Ω_r respectively for convenience, but we must not confuse them with the canonically named $\Omega_{B,rad} = \frac{\rho_{B,rad}}{\rho_{crit}}$. Having $a_{(t)}$ we can plot the values of these quantities

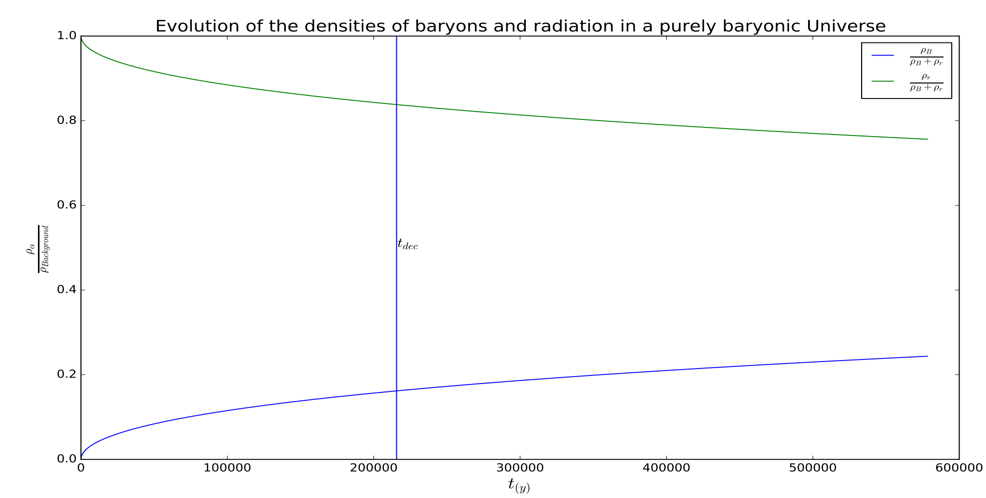

Figure 3: *Baryonic density is much smaller than the radiation density, but the former falls with a^{-3}, while the later does so with a^{-4}*

We can see that even going beyond the time of decoupling, the two ratios do not cross. This is a consequence of not having considered the DM, 6.5 times more aboundant that the baryonic matter. This result implies that, for the entire time of our integrations, radiation will dominate the evolution of the Universe and its inhomogeneities. Equation (28) becomes

$$\ddot{\delta}_{B,k} + 2\frac{\dot{a}}{a}\ddot{\delta}_{B,k} = \frac{3}{2}(\frac{\dot{a}}{a})^2(\Omega_B\delta_{B,k} + \Omega_r\delta_{r,k}) - \frac{1}{3}\Omega_r\frac{c^2}{a^2}k^2\delta_{r,k} \tag{30}$$

Now, for this equation only two quantities remain undefined: c and k. For the first one, one of the most famous constants in Physics, we only have to put it in the correct units:

$$c = 3\cdot 10^8[\frac{m}{s}] = 3.066\cdot 10^{-7}[\frac{Mpc}{y}]$$

where we used $1m = 3.24078\cdot 10^{-23}Mpc$ and $1y = 3.1536\cdot 10^7 s$. k will prove to be more resistant to proper determination. The wavenumber k is related to the radius R of a sphere than comprises a scale by the relation

$$k = \frac{2\pi}{R}$$

but, how can we determine the comoving distance R associated to a scale of mass M? We can see a derivation of this relation in chapter V in [2], but we will write it here in a slightly different, albeit equivalent, manner

$$M_{(R)} = 1.16\cdot 10^{12}\Omega_m M_\odot h^{-1}(\frac{R}{h^{-1}Mpc})$$

in units of $h^{-1}Mpc$. For convenience, we will take $h = 0.7$ and use Mpc as our units.

All elements in equation (30) are now defined. For the cases of only gravity and radiation presure, the relation

$$\dot{\delta}_{r,k} = \frac{4}{3}\dot{\delta}_{B,k} \Rightarrow \delta_{r,k} = \frac{4}{3}\delta_{B,k}$$

and we can put eq. (30) as:

$$\ddot{\delta}_{B,k} + 2\frac{\dot{a}}{a}\ddot{\delta}_{B,k} = [\frac{3}{2}(\frac{\dot{a}}{a})^2(\Omega_B + \frac{4}{3}\Omega_r) - \frac{4}{9}\Omega_r\frac{c^2}{a^2}k^2]\delta_{B,k} \tag{31}$$

expression where the sign of $[\frac{3}{2}(\frac{\dot{a}}{a})^2(\Omega_B + \frac{4}{3}\Omega_r) - \frac{1}{3}\Omega_r\frac{c^2}{a^2}k^2]$ will define whether or not we have oscillations. But we will dwell in this issue later on.

If we introduce photon diffusion we no longer have a simple relation between $\dot{\delta}_{rk}, \dot{\delta}_{B,k}$ and $\delta_{r,k}, \delta_{B,k}$, but this following formula

$$\dot{\delta}_{r,k} = \frac{4}{3}\dot{\delta}_{B,k}(\frac{2}{\sqrt{1 + 4(1.493\cdot 10^6[Mpc]ka^2)^2}})^{0.9} - \frac{\mu k^2}{a^2}\delta_{r,k}$$

Remember the definition of $\mu = \mu_{(a)}$

$$\mu_{(a)} = \frac{1}{3}\frac{1}{n_{e(a)}\sigma_T}c \tag{32}$$

with $n_{e(a)}$ the number density of electrons at a given a

$$n_e = \frac{0.218}{a^3}\frac{1}{m^3} = \frac{6.40531 \cdot 10^{66}}{a^3}\frac{1}{Mpc^3}$$

and σ_T the Thomson scattering cross-section for electrons

$$\sigma_T = 0.665 \cdot 10^{-28} m^2 = 6.9863 \cdot 10^{-74} Mpc^2$$

And with that we have defined the set of equations to solve and all the necessary parameters to do so.

$$\ddot{\delta}_{B,k} + 2\frac{\dot{a}}{a}\ddot{\delta}_{B,k} = \frac{3}{2}(\frac{\dot{a}}{a})^2(\Omega_B\delta_{B,k} + \Omega_r\delta_{r,k}) - \frac{1}{3}\Omega_r\frac{c^2}{a^2}k^2\delta_{r,k} \tag{33}$$

$$\dot{\delta}_{r,k} = \frac{4}{3}\dot{\delta}_{B,k}(\frac{2}{\sqrt{1 + 4(1.493 \cdot 10^6 ka^2)^2}})^{0.9} - \frac{\mu k^2}{a^2}\delta_{r,k} \tag{34}$$

If necessary, we will modify the value of σ_T manually to exhacerbate or diminish the effect of photon diffusion ($\sigma \uparrow \Rightarrow \mu \downarrow$, and viceversa).

After doing this, we will proceed to add DM to the mix. This only implies that now $\rho_B + \rho_{rad}$ is $\rho_B + \rho_{rad} + \rho_{DM} = \rho_m + \rho_{rad}$, having $\Omega_{0,DM} = 0.26$ and now $\Omega_{0,m} = \Omega_{0,DM} + \Omega_{0,b} = 0.3$, and the equations governing baryonic density fluctuations will be

$$\ddot{\delta}_{B,k} + 2\frac{\dot{a}}{a}\ddot{\delta}_{B,k} = \frac{3}{2}(\frac{\dot{a}}{a})^2(\Omega_B\delta_{B,k} + \Omega_{DM}\delta_{DM,k} + \Omega_r\delta_{r,k}) - \frac{1}{3}\Omega_r\frac{c^2}{a^2}k^2\delta_{r,k} \tag{35}$$

the relation that gives us $\dot{\delta}_{r,k}$ in relation to $\dot{\delta}_{B,k}$ does not change, and we have a new equation that governs the fluctuations of dark matter:

$$\ddot{\delta}_{DM,k} + 2\frac{\dot{a}}{a}\ddot{\delta}_{DM,k} = \frac{3}{2}(\frac{\dot{a}}{a})^2(\Omega_B\delta_{B,k} + \Omega_{DM}\delta_{DM,k} + \Omega_r\delta_{r,k}) \tag{36}$$

where the evolution of $\delta_{DM,k}$ will be affected by the behaviour of $\delta_{B,k}$ and $\delta_{r,k}$.

Apart from the changes on the equations, we will also have to pay attention to the change of values of $a_{(t)}$ and Ωs, the former being a consequence of the later. Eq. (26) remains valid, but $\Omega_{0,DM}$ is now $\neq 0$, thus changing $\Omega_{0,m}$ and, therefore, $a_{(t)}$.

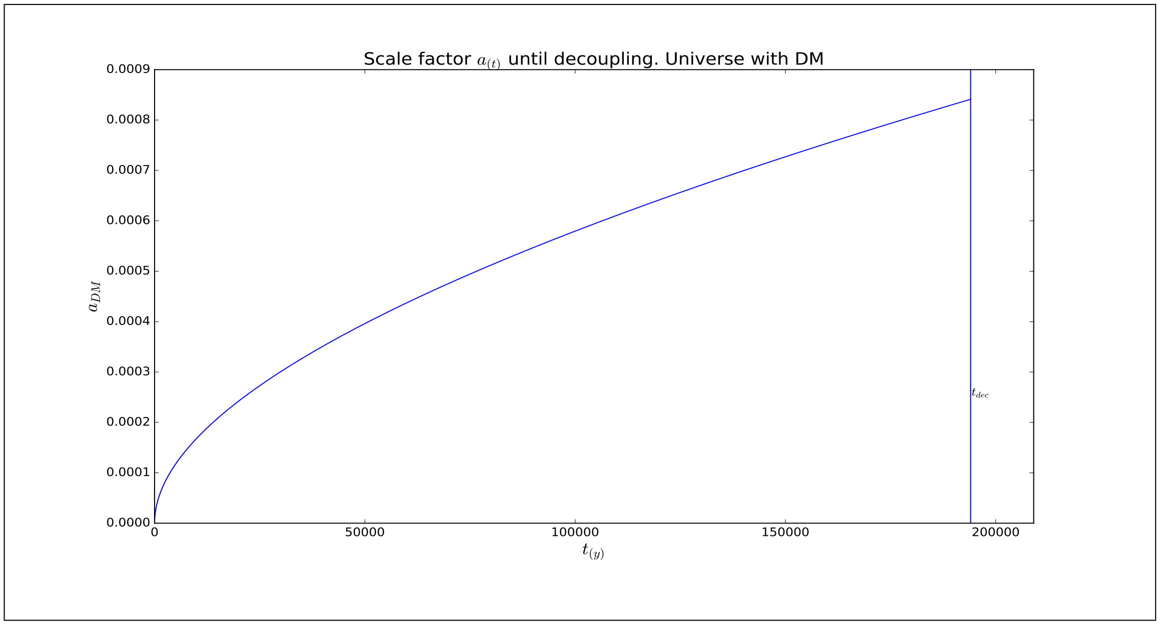

Figure 4: *At first sight, the form of* $a_{(t)}$ *remains more or less unchanged*

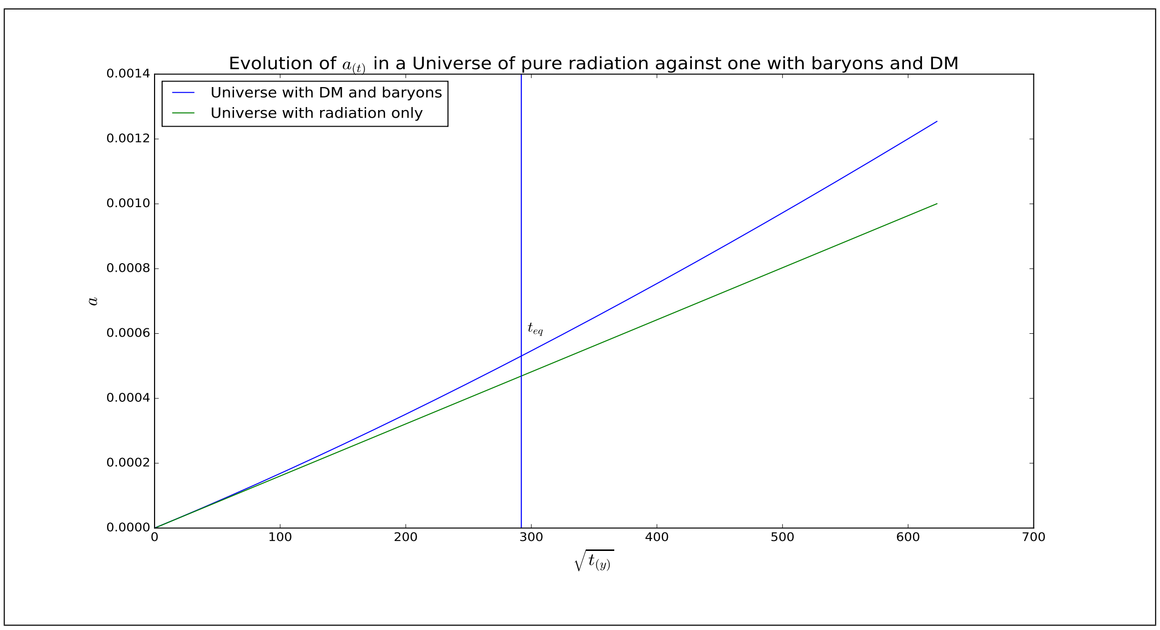

Figure 5: *But if we compare it with a for a radiation-dominated Universe, the departure is more evident than in the case without DM*

With decoupling taking place at $t_{dec} = 165494$. More important is the change on the ratios of matter and radiation against the background density. We now also have the quantity $\frac{\rho_{DM}}{\rho_B+\rho_{DM}+\rho_{rad}}$.

If we plot the three density ratios and the sum of the ones from baryonic matter and DM (ρ_M):

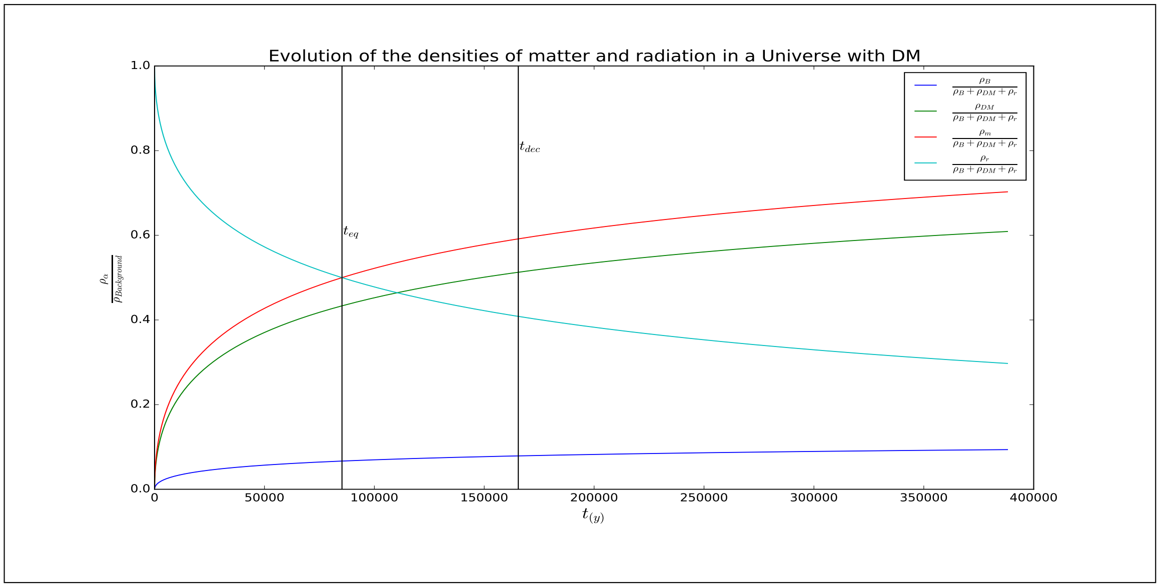

Figure 6: *Unlike the previous case, radiation stops dominating during the studied time.*

we can see that, at $t = 85280 = t_{eq}$ the radiations stops dominating and now matter dictates the evolution of the expansion.

Finally, we will take the values of δ for the desired species (baryons, DM, etc) at specific times for different scales to build the power spectrum.

2.2 Results

2.2.1 Purely baryonic Universe

We have detailed the obtention of the values of $a_{(t)}$ for a baryonic Universe in the previous section. We will probe each different case with two scales, $M_1 = 10^{13}M_\odot$ and $M_2 = 10^{14}M_\odot$. For these scales, we have an R of

$$\begin{aligned} R_1 &= 6.90Mpc \\ R2 &= 14.88Mpc \end{aligned}$$

and corresponding values of k

$$\begin{aligned} k_{(M_1)} &= 0.91Mpc^{-1} \\ k_{(M_2)} &= 0.42Mpc^{-1} \end{aligned}$$

The equation resulted from disregarding the effect of radiation pressure upon the baryon-radiation fluid describes a simple, gravitation-directed evolution

$$\ddot{\delta}_{B,k} = \frac{3}{2}(\frac{\dot{a}}{a})^2(\Omega_B + \frac{4}{3}\Omega_r)\delta_{B,k} - 2\frac{\dot{a}}{a}\dot{\delta}_{B,k}$$

that, when solved, produces the following results for M_1 and M_2:

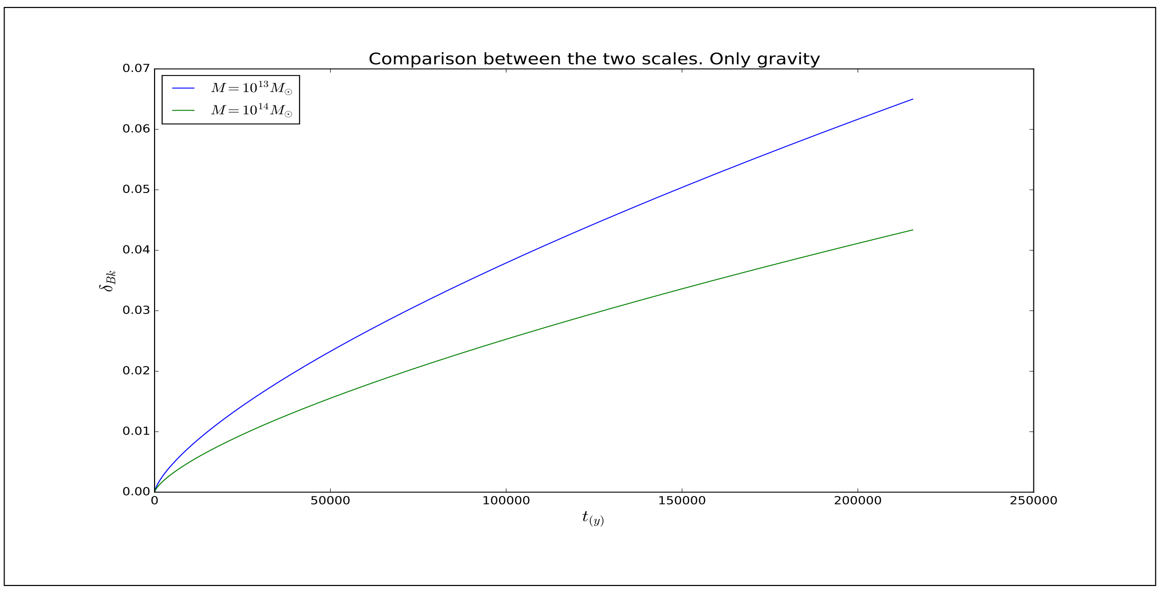

Figure 7: *The general shape of the evolution of different scales is the same when there is not radiation pressure*

The main implication of the kind of Universe proposed on the evolution of $\delta_{B,k}$ can easily be seen in the graphic when we compare both scales. What is happening there, then? If we take a closer look at the equation solved, we can see that there is no dependence on the masses or scales implicated in the inhomogeneities at all, only if we have different initial conditions for different scales (which happens, we took the initial spectrum to be the one of Harrison-Zel'dovich). δ is after all a dimensionless parameter, and it really does not contain information on the size of the inhomogeneity, only on the departure from the mean density of the specified species in the Universe. Therefore, every δ will evolve in exactly the same way, but parting from different initial values. This means that when plotting the power spectrum at the time of decoupling we will recover the Harrison-Zel'dovich spectrum, as we will see further down this section (remember that we will use the dimensionless $\Delta^2_{(k)}$, that is $\propto k^4$ instead of k^1). We can represent the evolution of $\delta_{B,k}$ respect the scale factor, instead of t

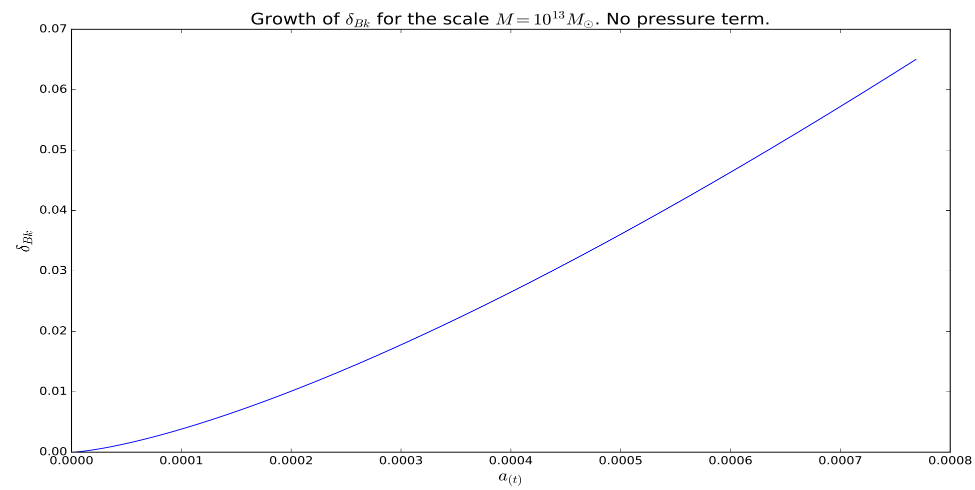

Figure 8: *Now we see how $\delta_{B,k}$ grows with a instead of t*

and see that $\delta \propto a^2$, something to be expected as we are in a radiation-dominated Universe for the entirety of the integration interval.

A scale-independent evolution, albeit easy to treat, is not a good aproximation at all to the real behaviour of δ (at least after our scales enter the horizon .We will focus on this statement in a moment). We will now introduce one of the two main complications (or allures, from a different point of view) of our treatment of inhomogeneities: the pressure due to the radiation part of our baryon-radiation fluid. We can remember equation 31

$$\ddot{\delta}_{B,k} + 2\frac{\dot{a}}{a}\ddot{\delta}_{B,k} = [\frac{3}{2}(\frac{\dot{a}}{a})^2(\Omega_B + \frac{4}{3}\Omega_r) - \frac{4}{9}\Omega_r\frac{c^2}{a^2}k^2]\delta_{B,k}$$

in which we now have the extra term $-\frac{4}{9}\Omega_r\frac{c^2}{a^2}k^2$. At a certain moment, this term will make the sign of the parenthesis accompanying δ to be < 0, which will make the equation one describing an oscillatory evolution. The pressure of the baryon-radiation fluid starts building up until a point is reached where this pressure is strong enough to oppose the compression due to gravity, deaccelerating it and eventually starting a process of decompression when $\dot{\delta}_{B,k} = 0$. This process does not stop at the point of equilibrium because our fluid has inertia, and continues decompressing beyond once reached equilibrium. This creates an underdense region with smaller associated pressure that will make the fluid to recompress again, thus creating the observed oscillation. We can visualize it

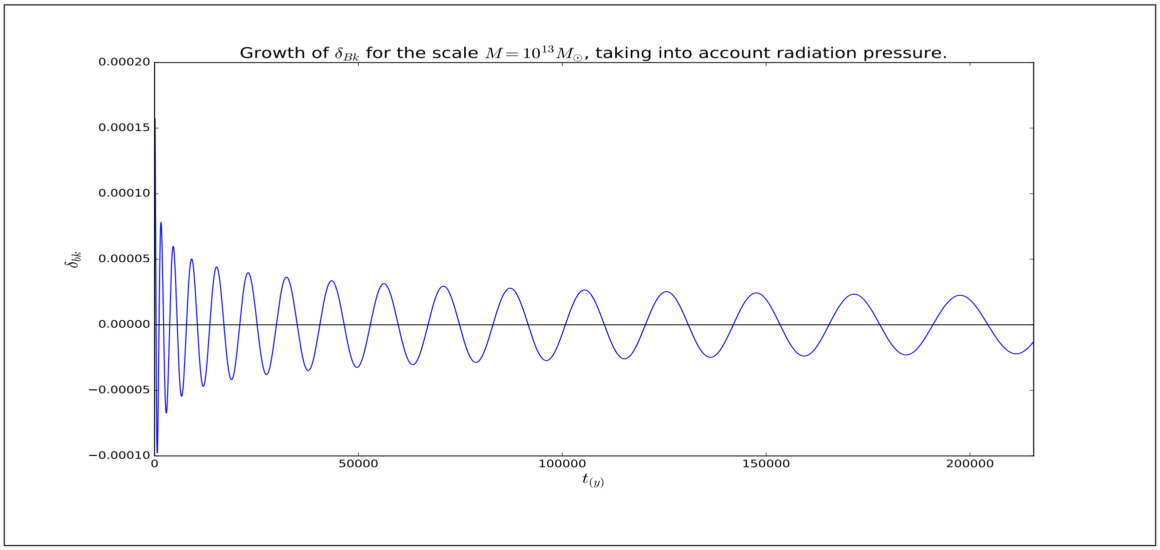

Figure 9: *There is a clear sinusoidal shape in the evolution of* $\delta_{B,k}$

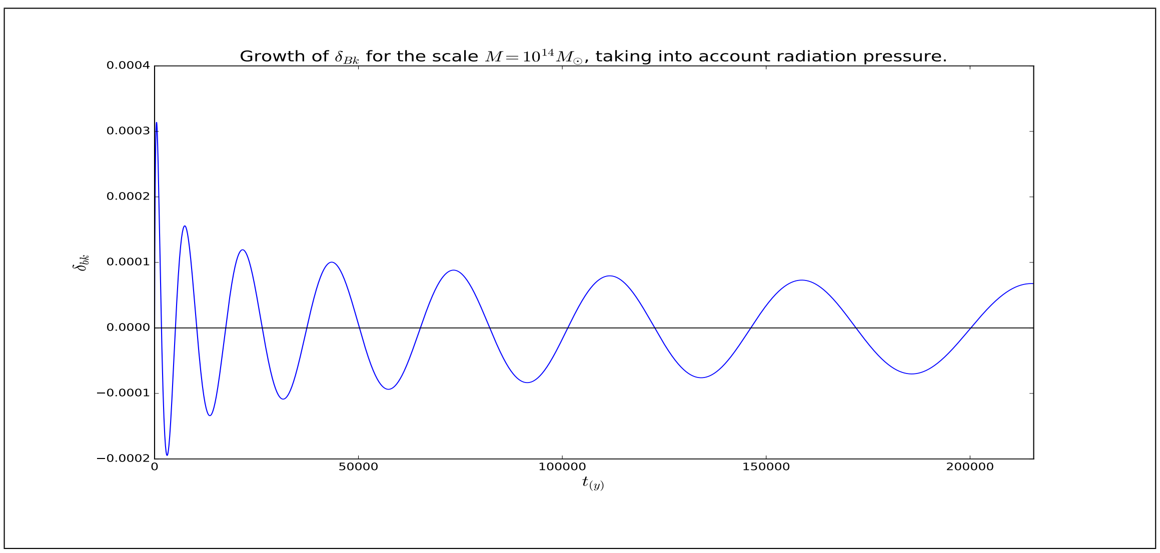

Figure 10: *The larger the scale, the bigger the amplitude and the smaller the frequency of the oscillation*

The change from the previous case is inmediately apparent. We now have an oscillation around an equilibrium that happens to be $\delta_{B,k} = 0$, with an absolute maximum at $t_{max} = 0.32 t_{Hor}$ in both cases. What does this imply? In the previous case, a small δ would keep growing, meaning than an overdense region would be more and more overdense as time comes. But with this results, we see that the growing of these δ is halted by the fact that baryons and photons are tighly coupled, and the pressure due to this last component is several orders of magnitude greater than the gas pressure of baryons (before recombination). Therefore, once the oscillations start until the point when they eventually stop, we have scales that have their growth frozen. Note that the peaks of amplitude are also significatively smaller that the values reached in the case of pure gravitational growth. We can see the two cases, pure gravitational growth and growth halted by pressure, to note how dramatic this effect is

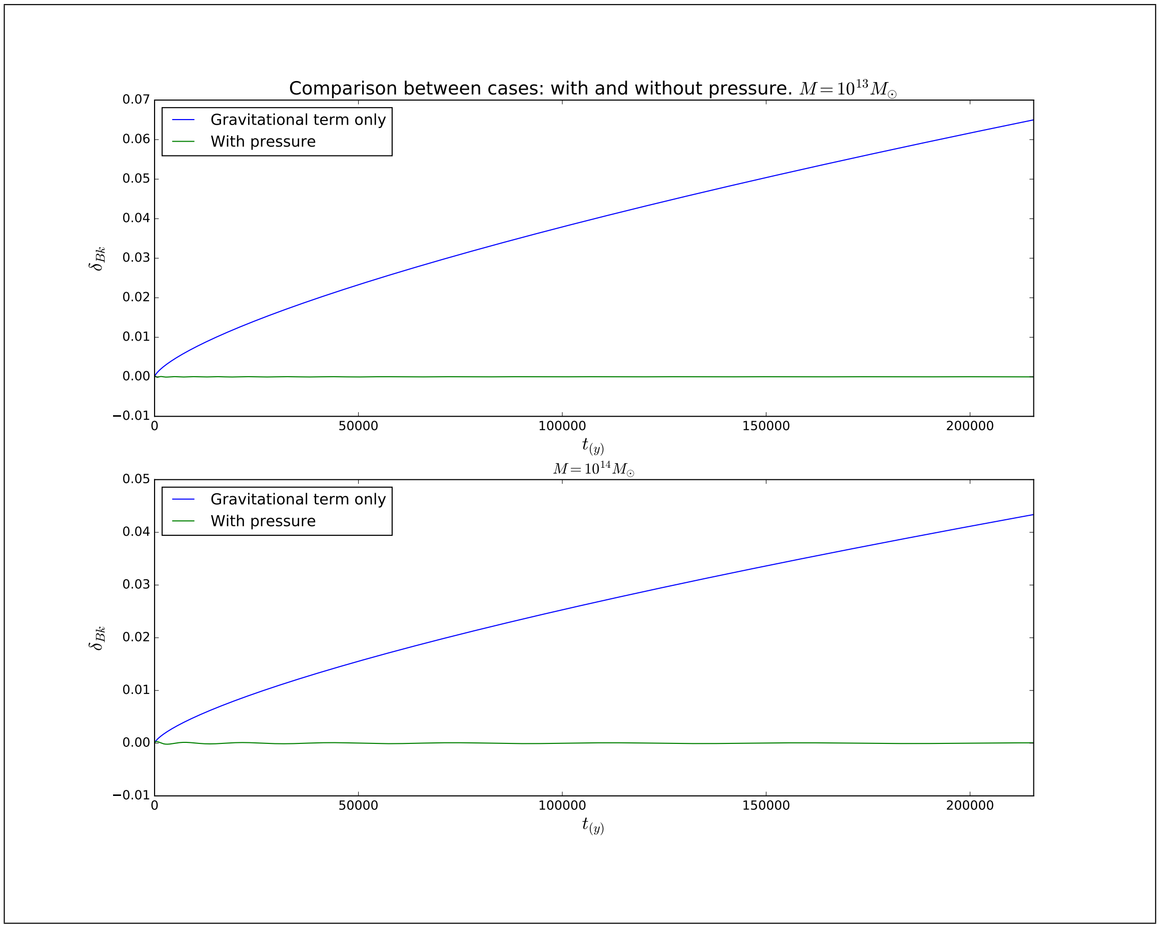

Figure 11: *$\delta_{B,k}$ gets effectively frozen whe radiation pressure acts*

We can compare now both structures

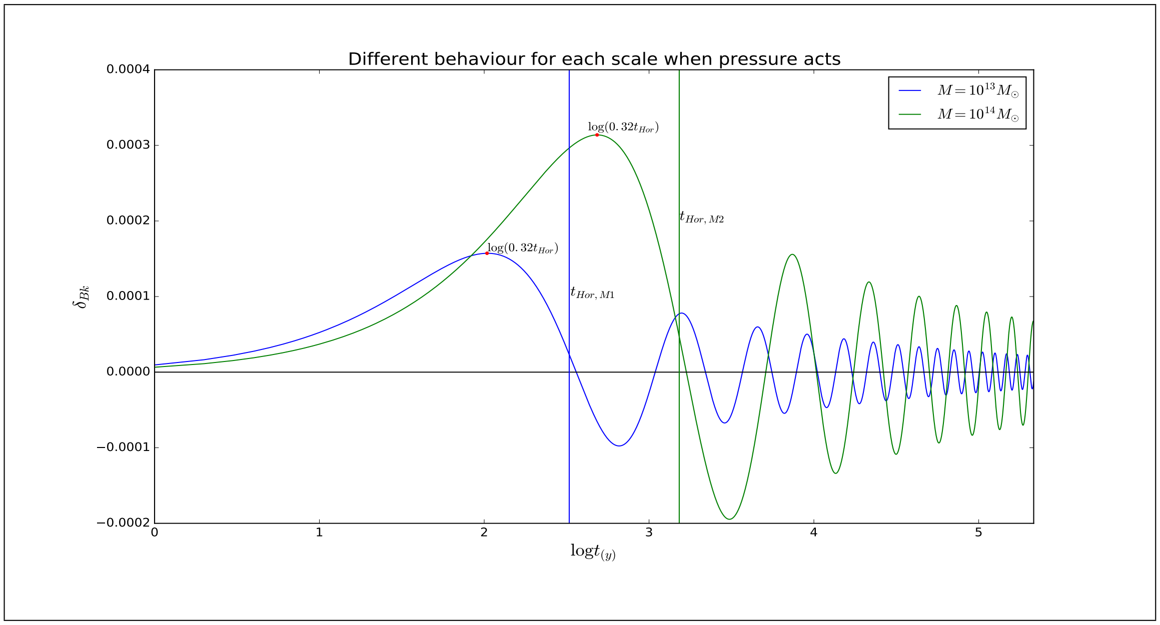

Figure 12: *Plotting against* $\log t_{[y]}$ *it is easier to see the complete evolution of inhomogeneities*

And we see more clearly now that we do not have the same behaviour for both: for a bigger scale we have a higher amplitude, and oscillations with smaller periods. This means that when the oscillation stops and the structures commence to grow again, they will do so starting from different initial conditions for δ than the ones given by the primordial Harrison-Zel'dovich spectrum and therefore the shape of this spectrum will not be maintained throughout time. We now can establish limits on different parameters by observing the values of δ for different scales (or related quantities, such as the power spectrum). It is also interesting to note the relation between $\delta_{B,k,max}$ with pressure and $\delta_{B,k,Hor}$ in the case of only gravity. $\delta_{max/grav,M1} = 0.235; \delta_{max/grav,M2} = 0.237$: fairly constant. The fall between $\delta_{B,k,max}$ and $\delta_{B,k}$ in the last peak is, for M_1, 0.143, and for M_2, 0.216. This quantities can be useful for the analytical treatment of the equations, for which it is necessary to assume the adiabatic approximation and where it can be applied. We have seen the general evolution of $\delta_{B,k}$ when subjected to radiation pressure, and we can see what happens to $\delta_{B,k}$ for times near t_{dec} but lets focus on the initial stages of its evolution. We can take as a significant time to study the early stages of this $\delta_{B,k}$ the time when each of the scales are inside the horizon. To compute said time we have two equivalent approachs: using the value of R_{Hor} or the value of a_{Hor}

$$a_{Hor} = \frac{2ct_{Hor}}{R_{Hor}}$$

Say that we use the value of R_{Hor}. We know the size of each scale, R_1 and R_2, and we have $a_{(t)}$. It is trivial then to find:

$$
\begin{aligned}
t_{HorM_1} &= 328y \\
t_{Hor,M2} &= 1525y
\end{aligned}
$$

(In the case with DM, those times will be $t_{HorM_1} = 86y, t_{Hor,M2} = 404y$). It is commonly stated that only when a scale enters the horizon it starts oscillating, because before that moment it cannot evolve as a whole, but if we study the earlier stages of our solution

we see that in fact the oscillations start before, at around $0.24t_{Hor}$ for each scale. We can write the equations being solved in a different way:

$$
\ddot{\delta}_{B,k} + 2\frac{\dot{a}}{a}\dot{\delta}_{B,k} = 4\pi G\rho_B(1 - \frac{1}{3}\frac{k^2c^2}{4\pi G\rho_B})\delta_{B,k}
$$

where

$$
\begin{aligned}
k_J^2 &= \frac{4\pi G\rho_B}{c^2} \\
\lambda_J &= \frac{2\pi}{k_J} \\
M_J &= \rho_b\frac{4\pi}{3}(\frac{\lambda_J}{2})^3
\end{aligned}
$$

and this equation starts having an oscillating behaviour once the term multiplying $\delta_{B,k}$ is negative. This happens for the observed time $0.24t_{Hor}$. In fact, the $\delta_{B,k}$ keeps on growing for a little bit after this time, until , the same moment when $\dot{\delta}_{B,k}$ becomes 0, at $t \simeq 0.32t_{dec}$. Remember that this is the moment when the pressure gradient is "stronger" than the gravitational potential, but the fluid was already growing before this moment. The compression then starts to decelerate, which translates into the observed ondulatory behaviour. Finally we are going to introduce the last ingredient on our mix for this Universe: photon diffusion. Photons, althought having a high scattering rate with electrons, still have a relatively long mean free path compared to electrons. When we have a compression, we are in a sense generating entropy, because we can expect photons to travel from the most overdense ("hot") regions to the colder ones (those outside the fluctuation). They can do so because of their longer mean free path. Remember the equation describing this phenomenon

$$
\dot{\delta}_{r,k} = \frac{4}{3}\dot{\delta}_{B,k}(\frac{2}{\sqrt{1 + 4(1.493 \cdot 10^6[Mpc]ka^2)^2}})^{0.9} - \frac{\mu k^2}{a^2}\delta_{r,k} \qquad (37)
$$

and compare it to the one we have without diffusion, $\dot{\delta}_{r,k} = \frac{4}{3}\dot{\delta}_{B,k}$. The new term depends on μ, defined in eq. (32), that at the same time is inversely proportional to the value of σ_T. The higher the value for the Thomson scattering cross-section, the softer the effect of photon diffusion: the photons wouldn't be able to escape the baryon-radiation fluid. But we do not have an arbitrarily high value of σ_T and some photons do get out of the baryon-radiation fluid. This produces a damping effect on the oscillations, named Silk damping after Joseph Ivor Silk (1968).The damping of oscillations due to diffusion is often explained as the scaping photons "dragging" the baryons with them, but this goes against the original argument that the photons that are able to scape are those that happen to not interact with baryons in the first place. We can explain the effect in a simpler way: when photons escape, the direct result is that we have fewer photons in the fluid.

Radiation being the dominant component of this Universe at the studied times, we are losing the main contributor to the radiation pressure (which dominates the oscillatory evolution) and also the gravitational evolution of $\delta_{B,k}$ and therefore damping both the radiation pressure and the gravitational evolution of the baryon-radiation fluid. We can observe how $\delta_{B,k}$ is going to behave under this newly introduced effect:

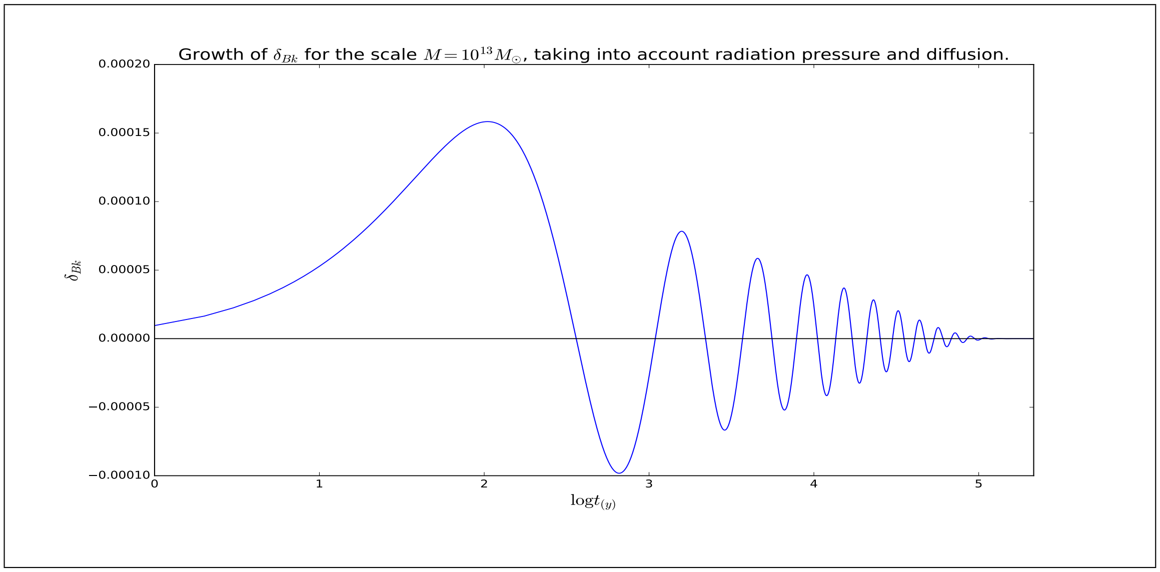

Figure 13: *$\delta_{B,k}$ for $M = 10^{13} M_\odot$ has practically vanished at the time of decoupling*

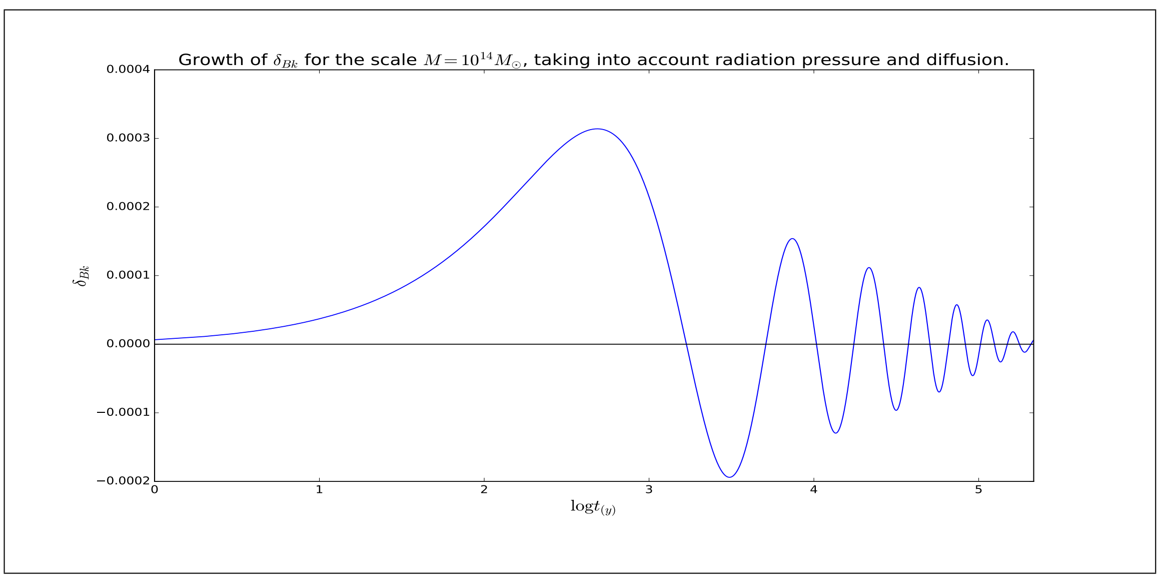

Figure 14: *The bigger scale is still present, but is almost negligible compared to the δ we would have without diffusion*

Also, it can be interesting to plot the values obtained for $\delta_{r,k}$ in this case, to see that when we approach the time of decoupling, this $\delta_{r,k}$ is no longer $\frac{4}{3}\delta_{B,k}$, but smaller. If we had not fixed the ionization ratio this effect would be even stronger approaching t_{dec}.

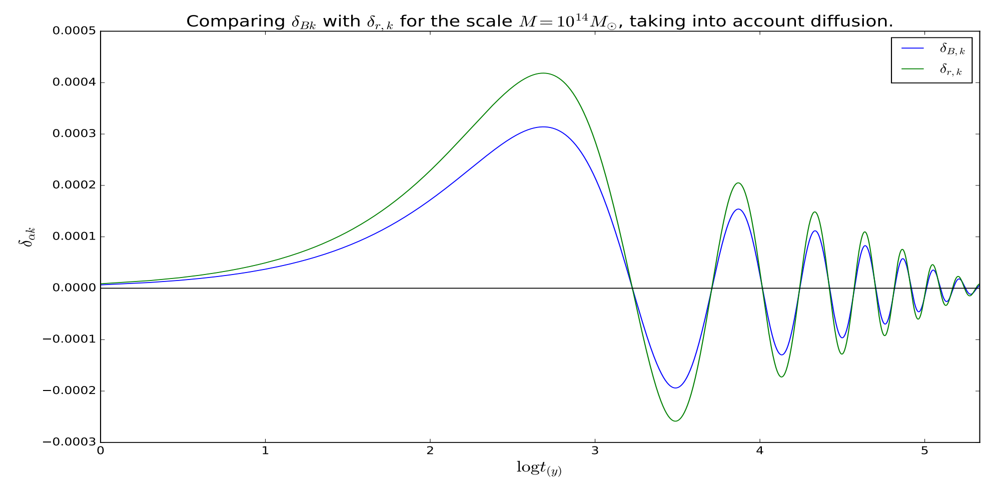

Figure 15: *As photon diffusion occurs, $\delta_{r,k}$ is no longer $\frac{4}{3}$ of its baryonic counterpat*

Comparing the evolution we have to the one obtained in the case of pressure without diffusion

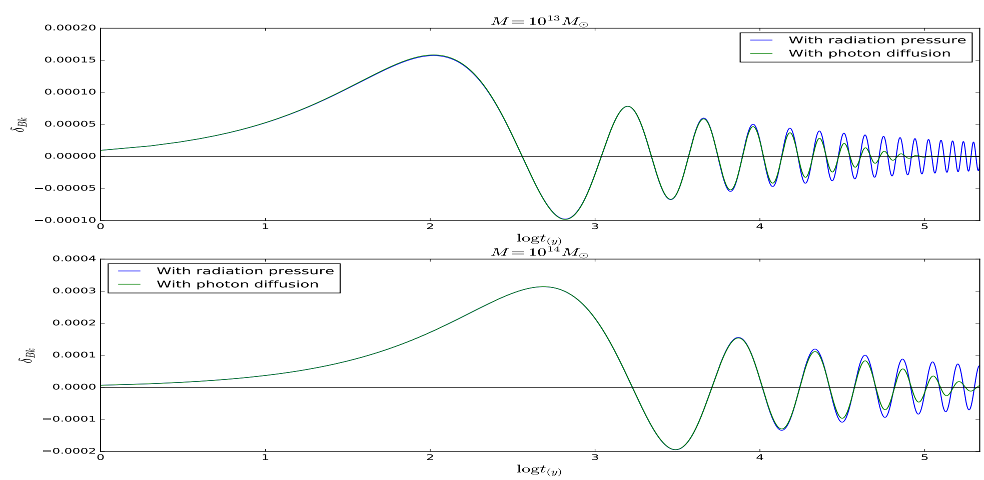

Figure 16: *We can appreciate the difference respect the case with only radiation pressure*

We can clearly note the damping of oscillations caused by diffusion. It is dependant on the size of the scale. Note that eq. (37) has a factor k^2. The higher the mass of the scale, the smaller $k_{(m)}$ and therefore the lower the damping. We can calculate the damping upon a scale: in the case of $M_2 = 10^{14} M_\odot$, we can expect the amplitude of the oscillation to be more or less 0.06 times the amplitude reached in the previous case where we did not take into account the photon diffusion, using the Silk approximate damping factor $e^{-(\frac{M_S}{M})^2}$, $M_S \simeq 1.67 \cdot 10^{14} M_\odot \equiv$ Silk Mass, defined below. In this case, our solution gives us a damping of 0.07 for this scale.

With scales smaller than a certain mass, named Silk mass or M_S (for which the amplitude of oscillations is reduced by a factor $1/e$ by the time of decoupling), we will have a stronger damping effect. This also supports the argument presented before that said that the reason for the damping is just the reduced density of photons: as we saw earlier, smaller scales have higher oscillation frequencies: for each cycle of oscillation we will lose photons, and the higher the number of cycles before decoupling, the higher the loss of photons. Another important factor that makes the damping of smaller scales to be greater is that they enter the horizon earlier, so they froze with an smaller size. As the mean free path of the photons would not change just because we are in less extense scale, it will be bigger in relation to the size of the lesser scales and the diffusion process will be stronger. We can compare both scales

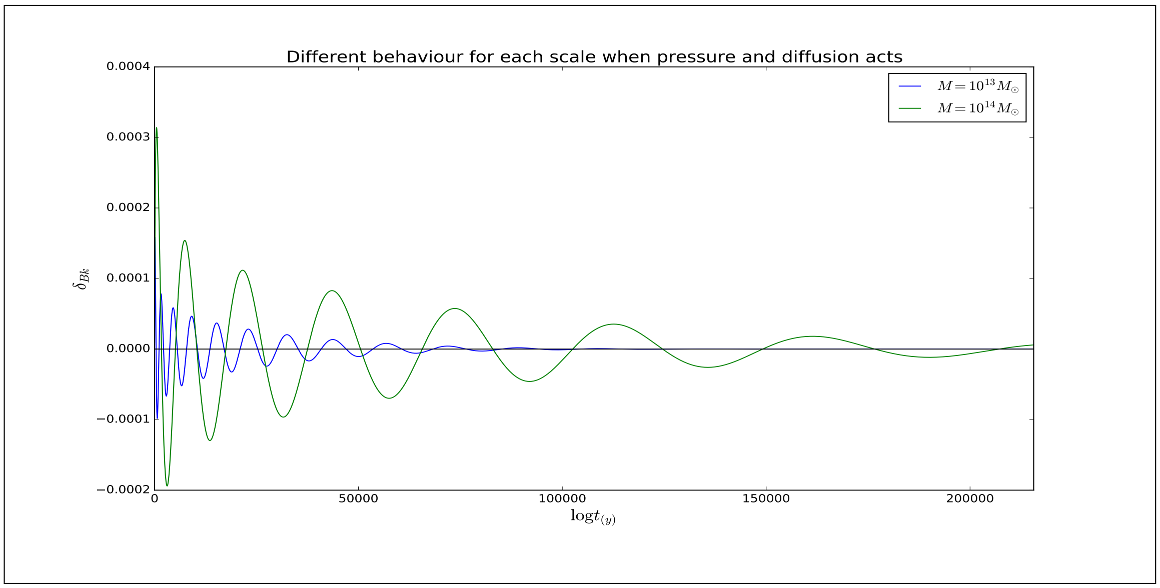

Figure 17: *The smaller scale, with smaller amplitude and higher frequency, gets wiped out earlier*

Photon diffusion affects primarly to radiation pressure, and this implies that, as happened before, they will start acting at the same time briefly before the scale enters the horizon. In addition, the damping has its strenght increased with time, but starts being almost negligible; as we saw comparing the cases with and without diffusion, at early times both cases are extremely similar. In any case, this behaviour will soon be dwarfed in comparison to the growth expected from the action of gravity alone.

Now that we have all the data, we can compute the power spectrum. For this we will consider δ^2. To do so, we only have to take the value of $\delta_{B,k}$ at the time of decoupling for various scales. Specifically, we will cover a range from $M = 10^{13} M_\odot$ to $M = 10^{16} M_\odot$ (value that may be larger than necessary: as mentioned before, structures as large as the Virgo supercluster are estimated to be one order of magnitude smaller than our biggest scale here). This corresponds to $k \in [0.18, 1.78]$. Remember that for bigger masses, k is smaller. We already noted that, in the case of just gravitational growth, all scales would grow in exactly the same fashion and keep the shape of the primordial power spectrum found at the end of the inflationary epoch. It is not surprising, then, that if we compute $P_{(k)} = \Delta^2_{(k)}$ (from eq. (20)) it will have:

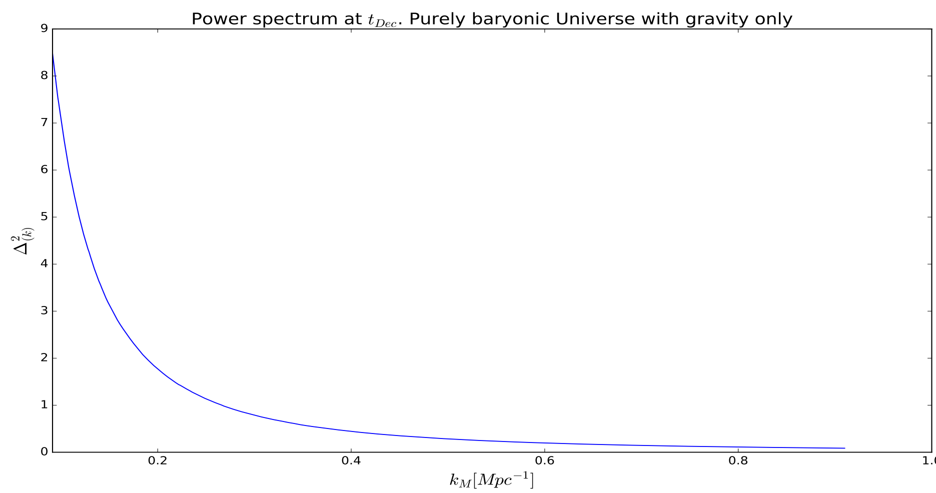

Figure 18: *Spectrum with the shape of Harrison-Zel'dovich*

But if we obtain the spectrum observationally, it does not maintain the primordial shape at all, indicating that there are more factors at play in the evolution of $\delta_{B,k}$ than gravity.

If we introduce pressure, we will have the following values

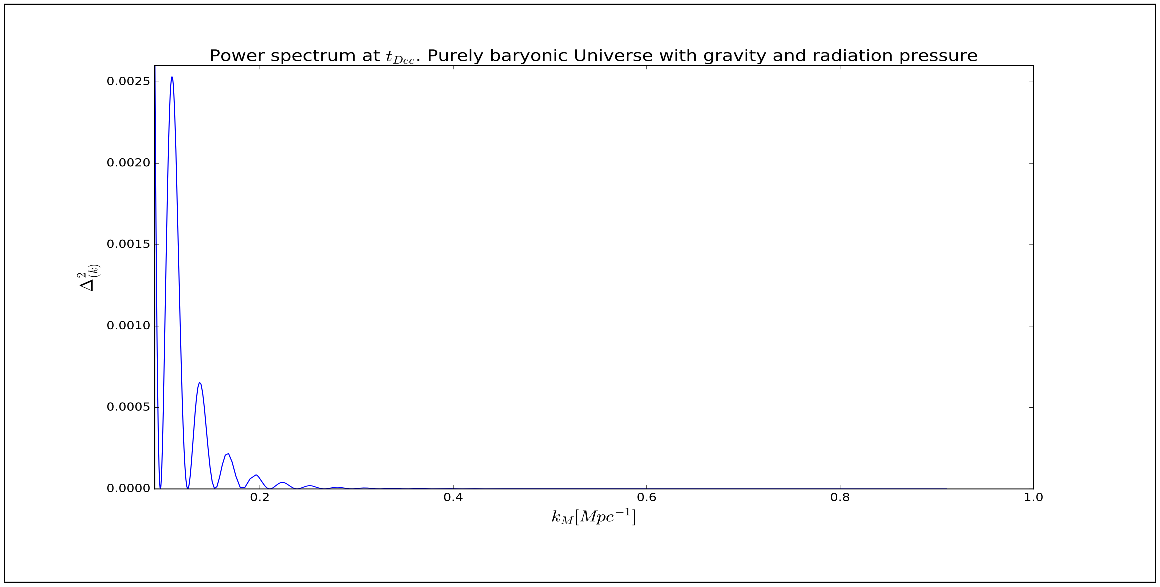

Figure 19: *The larger the scale, the earlier the time they "froze". Smaller scales did not grow and this translates to* $P_{(k)}$

The smaller scales entered the horizon earlier: that is, their growth froze at an earlier time, with a smaller amplitude of δ_B and consequently reached the time of decoupling with a smaller amplitude. If we now introduce photon diffusion, the scales smaller than $M_S \simeq 6.93 \cdot 10^{14} M_\odot$ had their amplitude wiped out and disappeared from the spectrum

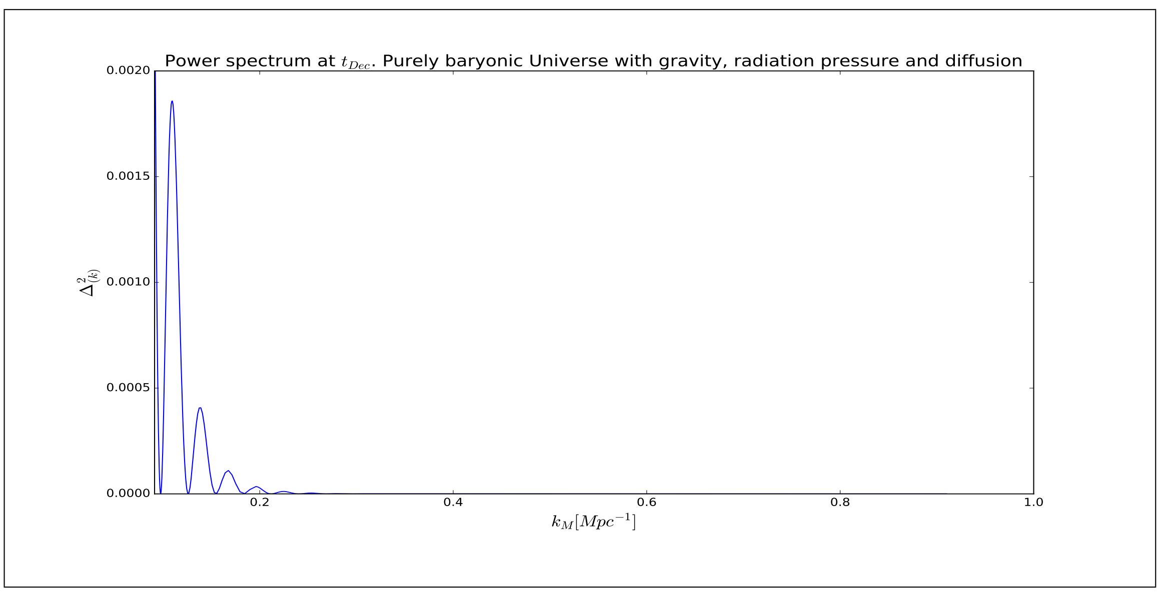

Figure 20: *On top of being frozen, smaller scales are wiped out due to photon diffusion*

We can overlap both spectrums to observe the actual effect of diffusion on smaller scales

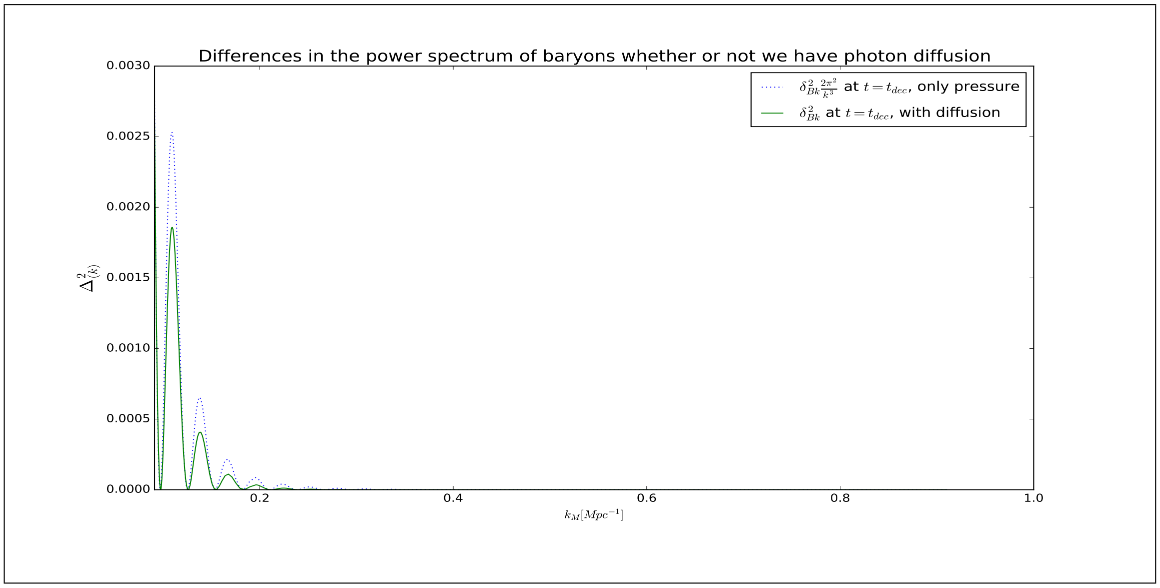

Figure 21: *Diffusion dampens the growth of all scales, and erases the ones smaller than M_S*

2.2.2 Universe with DM

We will now introduce one more ingredient to the mix. So far, the only components in our Universe have been baryonic matter, radiation (in the form of photons) and curvature (this last element being only introduced for completeness, it will be negligible for all t). This treatment can be convenient to work with, but it lies far away from the observed parametres of the Universe. Not taking into account, for example, Λ is not that big of a problem, since it will be negligible for $t < t_{dec}$ and beyond, but, if we recall the results of computing the density ratios of our components, the introduction of DM implies that the time of radiation-matter equality comes much sooner than in a baryonic Universe, where $t_{eq} \simeq 578258y$ instead of $t_{eq} \simeq 85280$ in the case of DM. If we had a radiation-dominated Universe for all $t < t_{dec}$ before, now a big chunk f time will be spent in a matter-dominated Universe. The particular aspects of the evolution treated in the previous section arise from the behaviour of $\delta_{r,k}$. If this component of the Universe is now non-dominant for some t we can expect the evolution of the different δ's to adapt to the new situation. Not only that, but the main contributor to the matter density, the DM (6.5 times more aboundant than baryonic matter) does not interact with radiation other than gravitationally. We will have two decoupled fluids, one consistent on baryonic matter and photons, and the other of dark matter (specifically, CDM: we consider the DM non-relativistic).

Having said this, we still have a situation that does not change significatively: the growth due purely to gravitation. The addition of DM does not carry any new dependence on the size of the scale for the gravity term, and we will find again that there is no difference in the evolution of inhomogeneities for different scales.

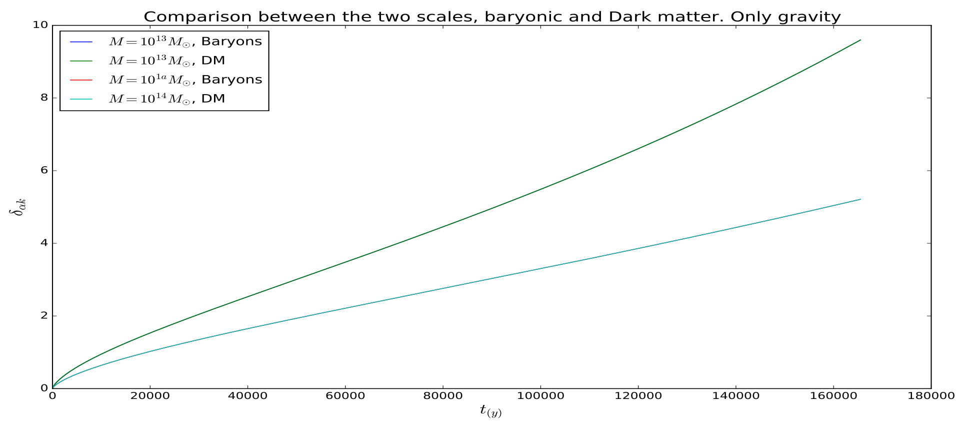

Figure 22: *The change from a radiation-dominated Universe to a matter-dominated one can be appreciated in the growth of δ*

We find again a simple and not very interesting evolution, although we have to note that the values of δ for both baryonic and Dark matter are the same for each scale: same inital conditions for the two matter components of a scale, coupled with the independence of the evolution of said initial condition with the scale or species we are dealing with, explains this situation. We can also appreciate how the evolution of the scale factor a affects the growth of δ: we are not in the realm of radiation-domination for the entirety of the studied time anymore, and this translates into the solutions of the equation. It is when we introduce the pressure term that we have in eq. (35), where we still have $\delta_{r,k} = \frac{4}{3}\delta_{B,k}$, when things get more interesting. Solving the equation produces the following results:

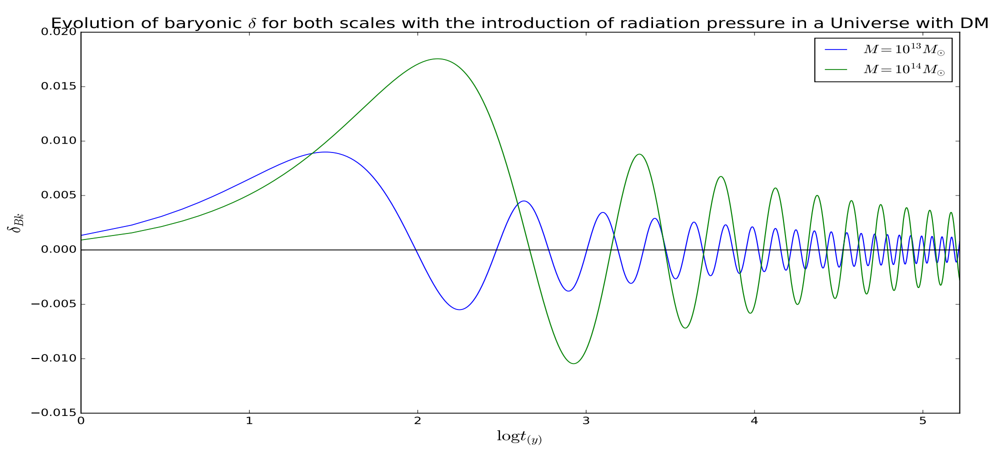

Figure 23: *Evolution very similar to the one without DM*

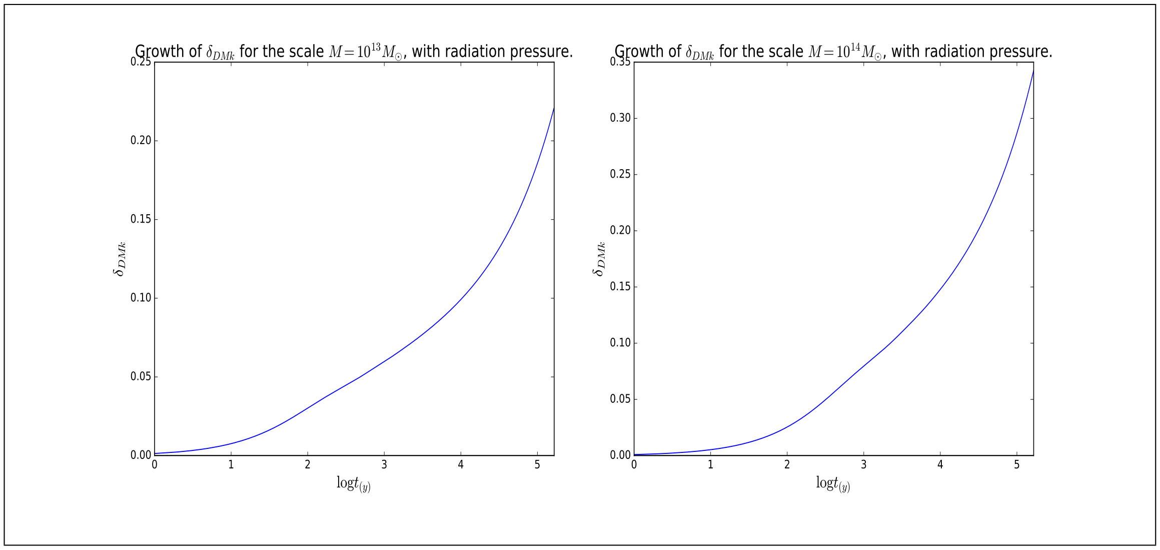

Figure 24: *DM appears to keep growing even with* $\delta_{B,k}$ *and* $\delta_{r,k}$ *oscillating*

The first figure shows the evolution of $\delta_{B,k}$, very similar to the previous case, and the second one show the evolution of $\delta_{DM,k}$. The dark matter behaves differently due to its non-interactive nature with radiation, and it does not get stuck in an oscillation as the other components. However, we can see some very small oscillating features in its evoution. Why? The DM is not coupled with photons, but it is still affected gravitationally by them. The term $(\Omega_B \delta_{B,k} + \Omega_{DM}\delta_{DM,k} + \Omega_r \delta_{r,k})$ in eq. (36) has the contribution to the gravitational potential of all species, including radiation, which dominates for a long while. Even after t_{eq} its contribution is large, being also exhacerbated by the addition of $\Omega_{B,k}\delta_{B,k}$, with $\delta_{B,k}$ coupled with $\delta_{r,k}$. DM will see its evolution repressed by the fact that both the radiation and the baryonic matter are "frozen" around $\delta = 0$. We will see this behaviour in the next images; what we see in fig. (24) is another effect of this oscillation of baryons and photons: if $\delta_{B,k}, \delta_{r,k}$ are oscillating, the potential depending on them will do the same. The gravitational potential affecting DM should oscillate, very very lightly, but oscillate nonetheless. This translates into this minor oscillation that will not halt the slight growing shape of the evolution of $\delta_{DM,k}$.

Let us compare the evolution of $\delta_{DM,k}$ with and without the influence of radiation pressure:

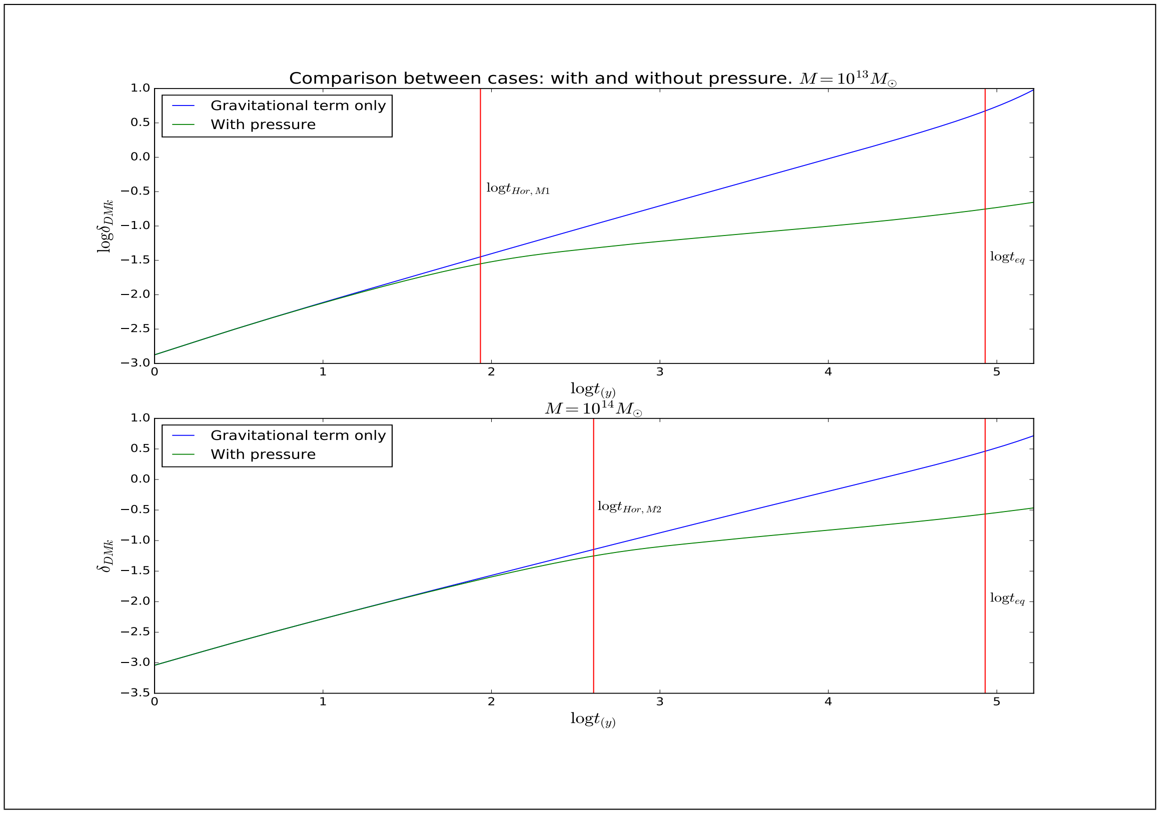

Figure 25: *Coincident at early times, around* t_{Hor} *for each scale, the growth of* $\delta_{DM,k}$ *freezes compared to the one in the pure gravitational case, but starts growing again under its own gravitation after* t_{eq}

The growth of $\delta_{DM,k}$ is seriously affected by radiation pressure (better said, by the lack of growth of $\delta_{B,k}, \delta_{r,k}$, consequence of the oscillations caused by radiation pressure) but, is it as "frozen" as $\delta_{B,k}$? The answer is no

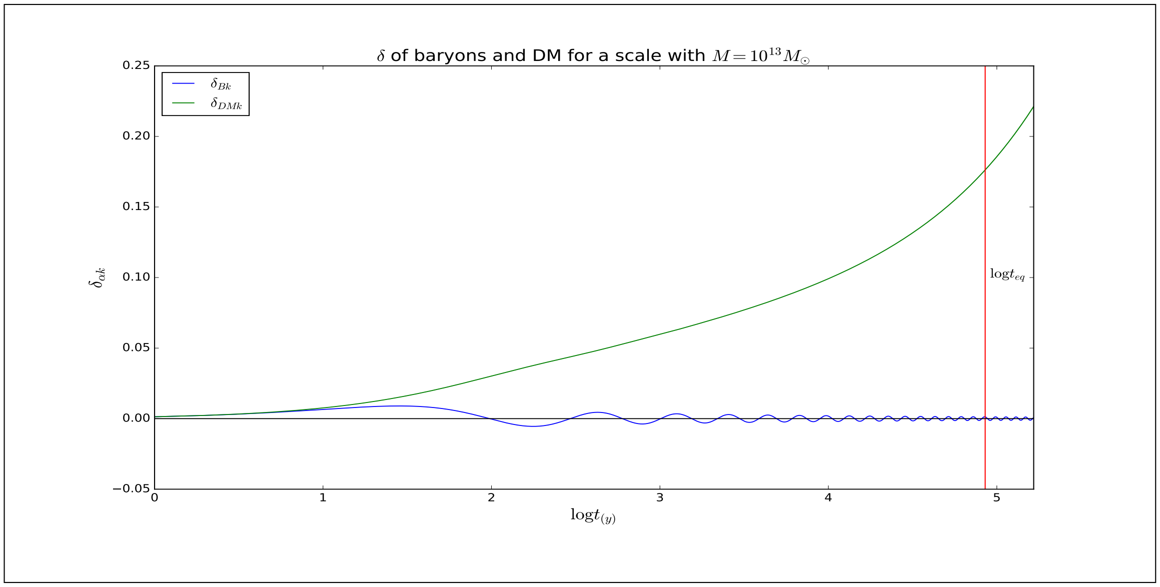

Figure 26: *$\delta_{DM,k}$ grows when compared to $\delta_{B,k}$*

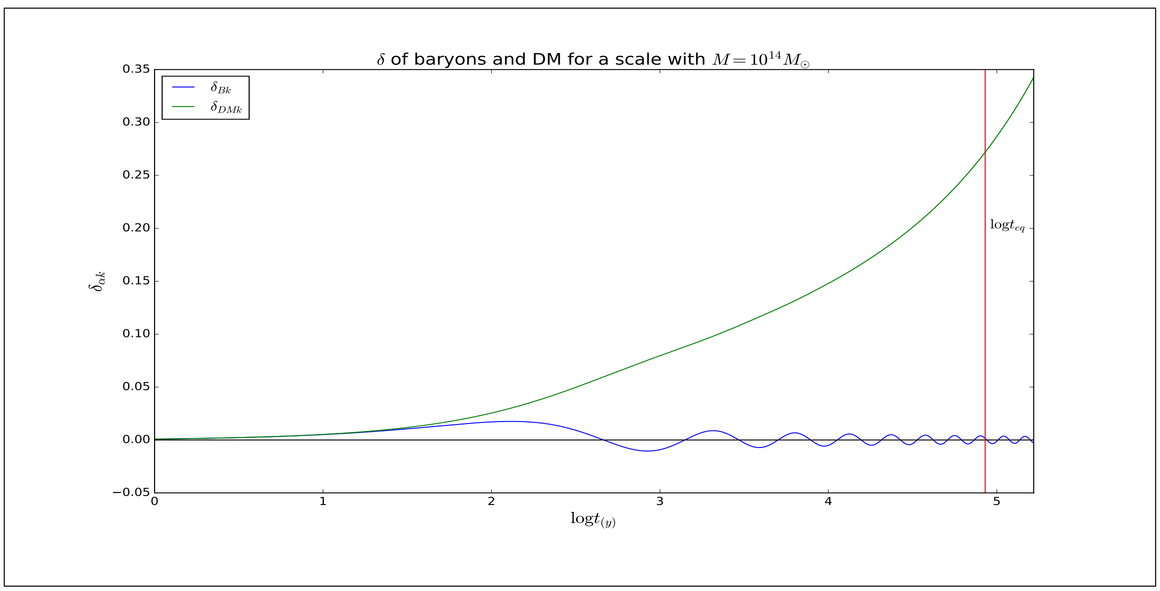

Figure 27: *DM exhibitis the same behaviour for both scales*

$\delta_{DM,k}$ grows more than $\delta_{B,k}$ (in accordance with the calculations carried in section 4.5 in [5], $\delta_{DM,k}$ grows by a factor $\simeq \frac{5}{2}$ during this stalled expansion), but still much less than what it would grow under the exclusive influence of gravity. As happened in the analysis of a purely baryonic Universe, the oscillation does not start inmediately but after some time near t_{Hor} for each scale. Now we introduce photon diffusion. The effects of radiation pressure upon $\delta_{B,k}$ will lose importance due to this phenomenon, exactly like in a purely baryonic Universe. There is a difference: now we have a DM component that kept growing, albeit very slowly, during the baryonic oscillations, coupled with the fact that after t_{eq} radiation no longer is the dominant element of the Universe, but matter, which is mainly comprised of the ever-growing DM. It is then not a surprise to see the following evolution of $\delta_{B,k}$:

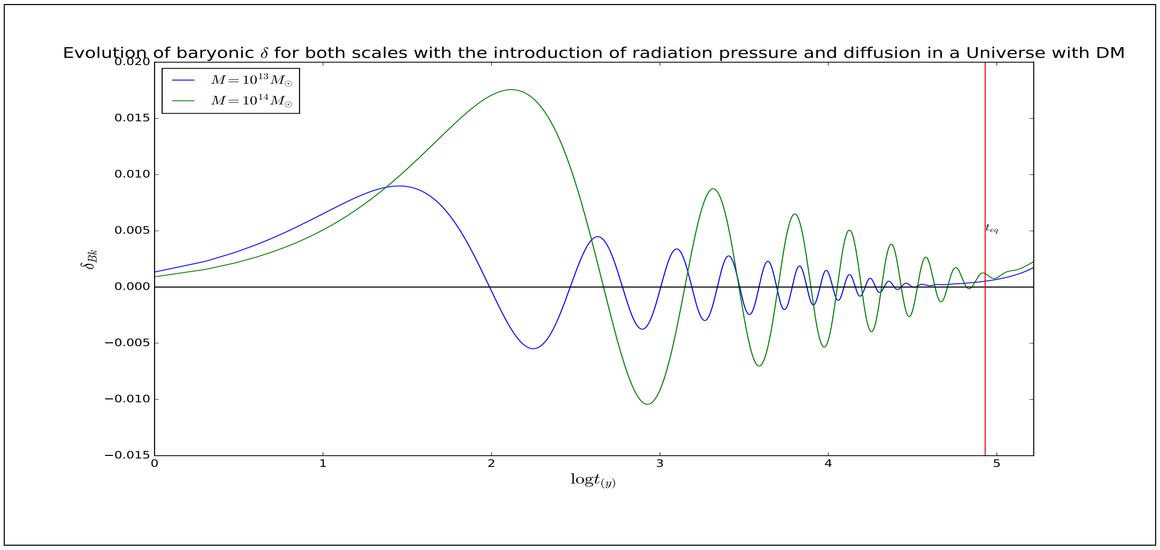

Figure 28: *Both scales are not oscillating around $\delta_{B,k} = 0$ for times near t_{eq}, even earlier in the smaller scale*

We can appreciate that around t_{eq}, $\delta_{B,k}$ starts growing again under the influence of DM, that has both a bigger Ω and a bigger $\delta_{DM,k}$, as we can see when we plot both species at the same time

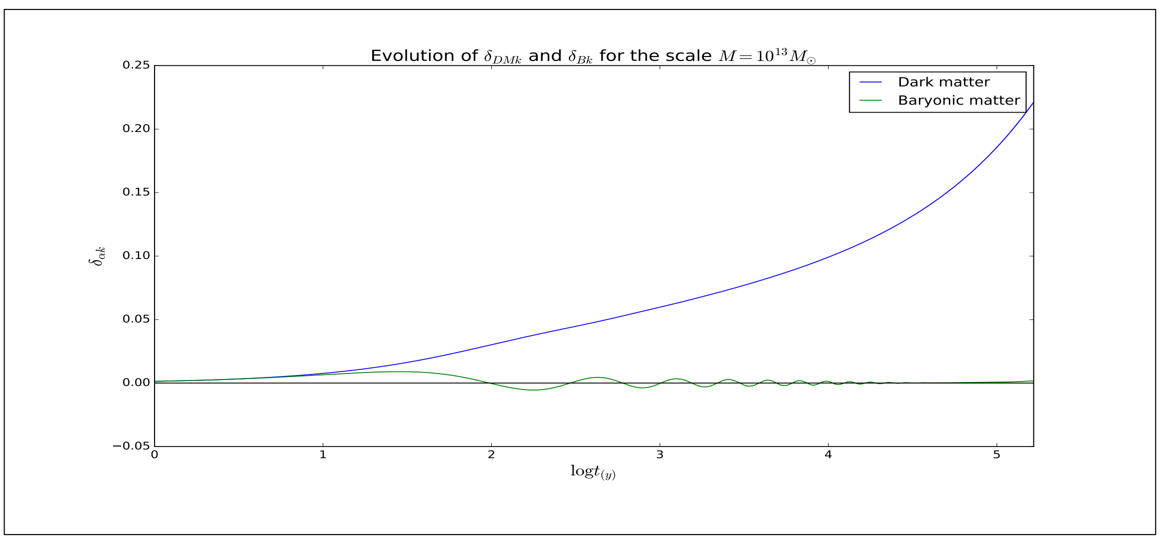

Figure 29: *With diffusion, $\delta_{DM,k}$ has a very similar behaviour to the one without it*

And essentially the same result for $M = 10^{14} M_{\odot}$

Basically, this means that photon diffusion accelerates the domination of DM in the evolution of inhomogeneities, apart from erasing small scales. The evolution of $\delta_{B,k}$ will be different from the one we had in the case of no diffusion, but it is still possible to state that until decoupling, the inhomogeneities are frozen when we compare them to the ones we would have if there was no pressure or photon diffusion. Again, we can study the time t_{Hor} for both baryons and DM and see that DM grows the same way up until the point when baryon-radiation oscillations starts, moment when said growth stagnates without a clear-cut difference between the cases with and without diffusion.

We now are in position to compute the power spectrum at t_{dec} in the same fashion as done for a baryonic Universe. We will see that, just as before, the gravitational evolution conserves the primordial shape of the spectrum for both DM and baryons

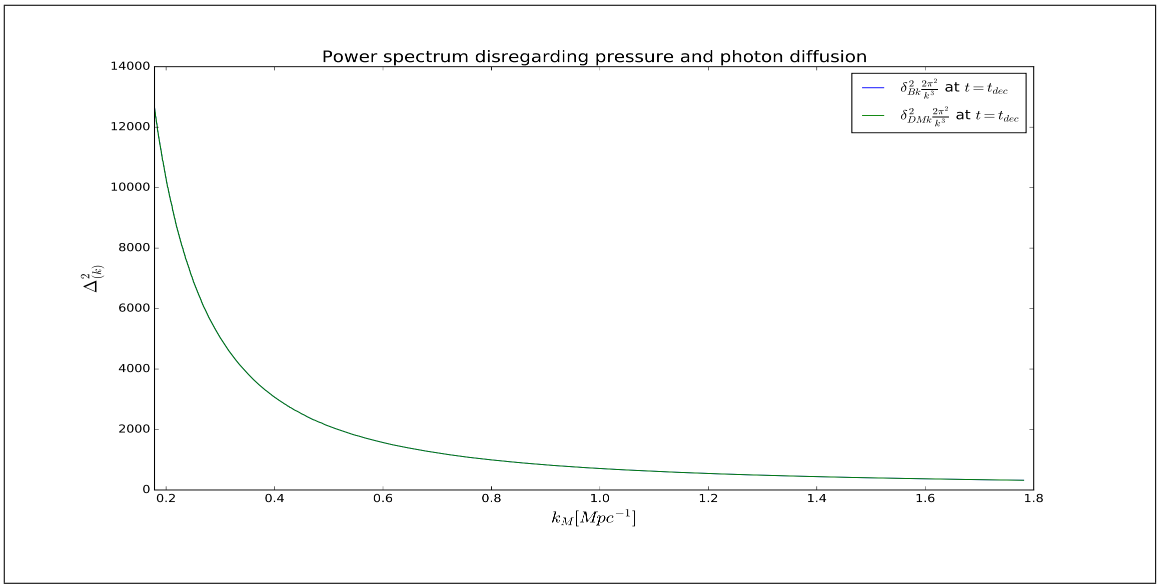

Figure 30: *Again, when we only have gravity, the shape of the primordial spectrum is conserved*

And, with same initial conditions of δ for the two kinds of matter, they end up with the same power spectrum. Now, if we add pressure, the baryonic power spectrum will be drastically changed, as a consequence of the coupling between radiation and baryonic matter:

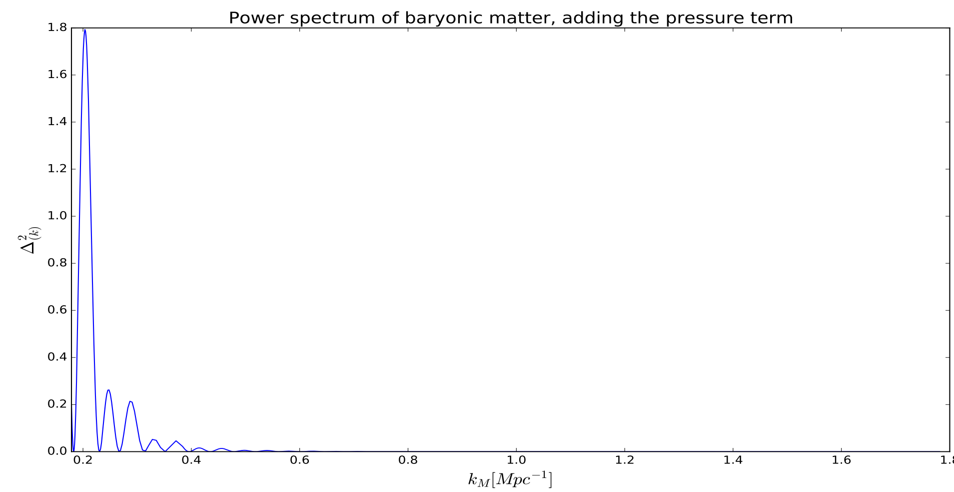

Figure 31: *The shape of this spectrum is essentially the same of the one of a Universe without DM, but with slightly higer values of* $\Delta^2_{(k)}$ *in general*

We have different behaviours for different scales. The smaller scales went into the horizon before, so it is expected for them to reach t_{dec} with smaller amplitudes.

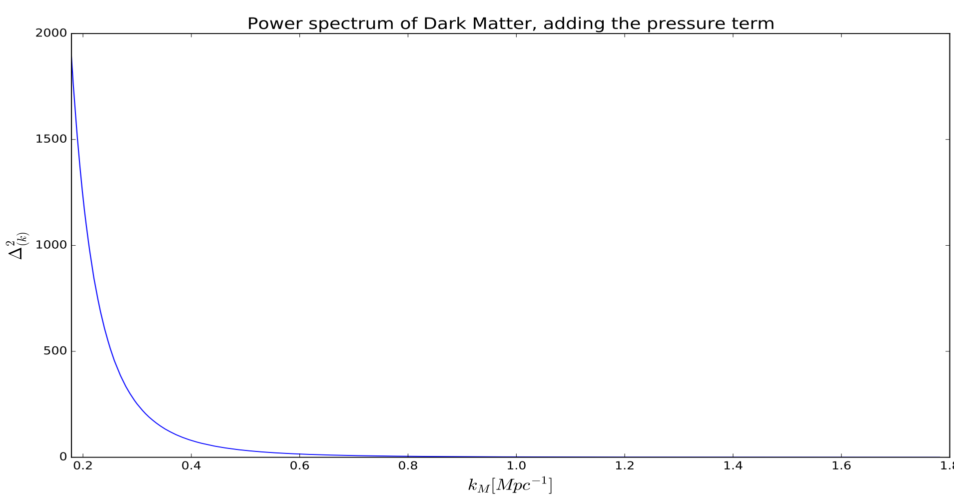

Figure 32: *Bigger scales behave as they would do without pressure, as they did not have time to freeze. Smaller ones are almost negligible compared to those*

Even if it does not oscillate, DM sees its growth interrupted by the baryonic oscillations. They start at different times, which means that δ_{DM} for bigger scales would have more time to grow before being interrupted. More importantly, the rest of the contributors to the gravitational potential (baryonic matter and radiation) are frozen. However, DM is not directly affected by radiation pressure, and its evolution is still defined by the gravitational potential: therefore, we will maintain the general shape of the primordial spectrum. The smoothness of this spectrum is consequence of the lightness of the transference of oscillations from δ_B to δ_{DM}: the dark matter power spectrum is much more smoother that the one for baryonic matter, something that contributes to the formation of structures that otherwise would not have had the time to form from decoupling until now.

If we add photon diffusion, what will happen is that smaller scales (higher k) will be wiped out. However, unlike in a purely baryonic Universe, δ_B will start growing again under the influence of the DM, so the power spectrum will not go to 0 in the wiped out scales as rapidly as in the baryonic case.

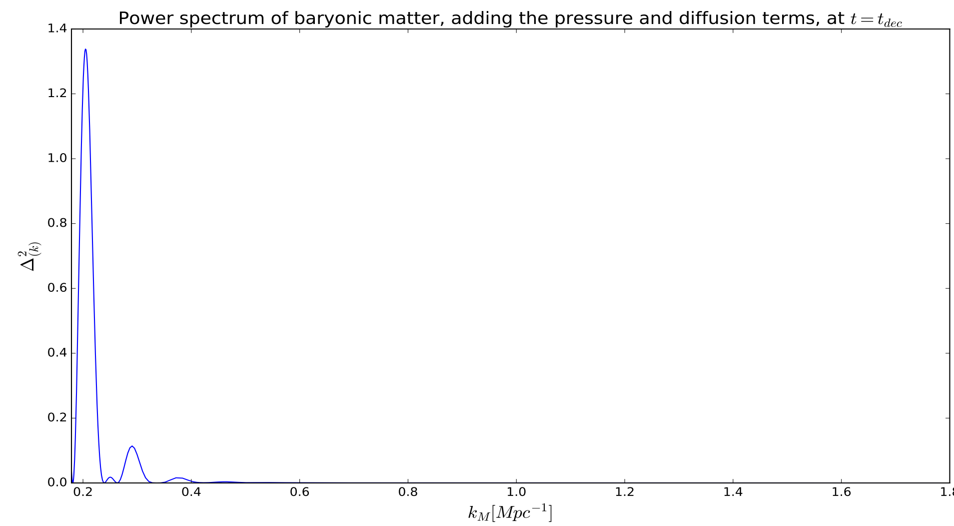

Figure 33: *We lose again the smaller scales*

Comparing the cases with and without diffusion

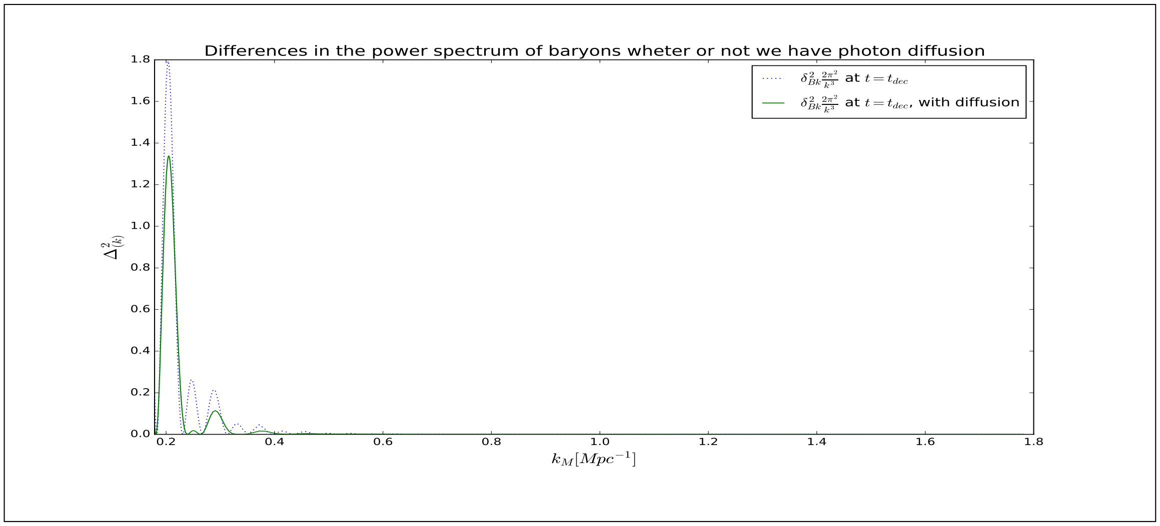

Figure 34: *Fewer and smaller peaks*

The peaks corresponding to the smaller scales are lost.

The DM power spectrum won't be significatively altered We can also compare the obtained

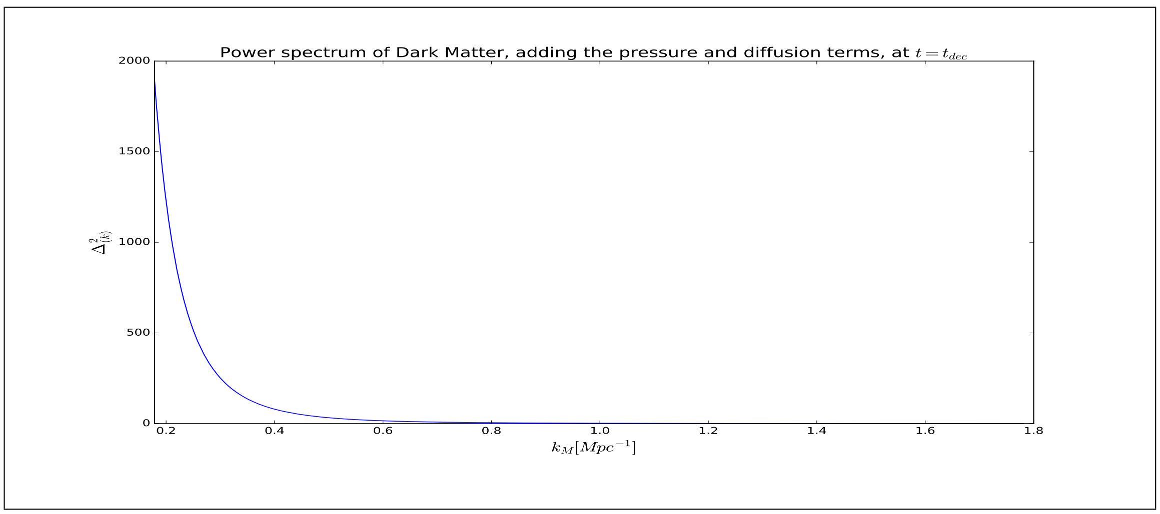

Figure 35: *Essentially, the same $P_{(k)}$ than in fig. (32)*

power spectrum at t_{dec} with the one we get when we go to bigger redshifts. We assumed that decoupling happened around $z = 1300$, now we are going to go beyond that, to $z = 1500$. It is not necessary to illustrate the differences in $P_{(k)}$ for DM: it has the exact same shape, only with slightly lower absolute values, given that the inhomogeneities would have had less time to grow. The most interesting case is the one of baryonic matter under the influence of radiation pressure and photon diffusion

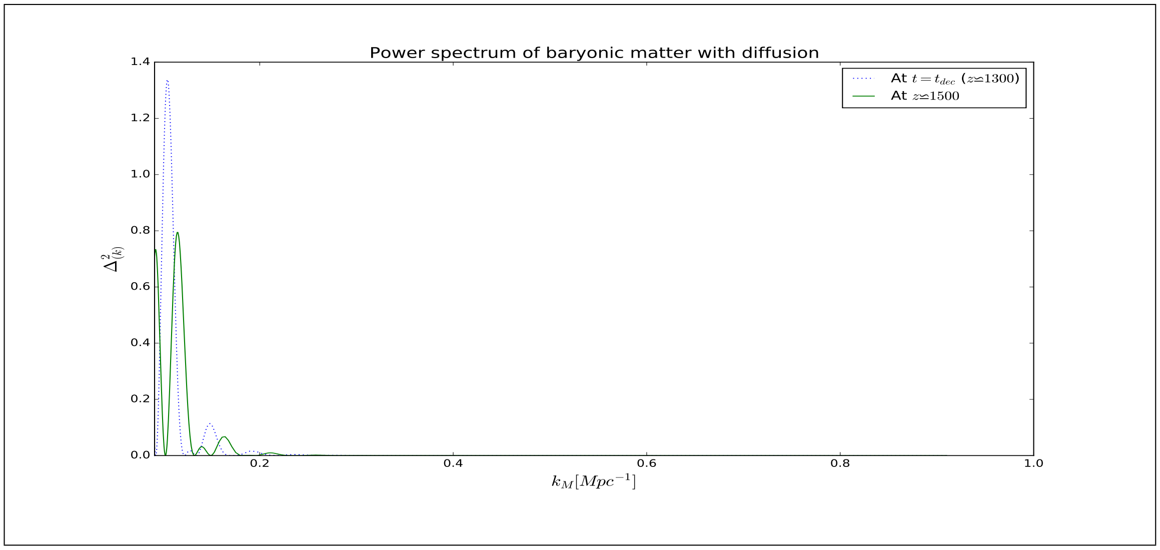

Figure 36: *At times earlier than t_{Dec} there has been less damping of small scales, and also the still present peaks have not grown so much under DM's influence*

Where we see that, apart from a general reduction of the values of $P_{(k)}$ for the same reason of having less time to grow (remember that before decoupling the presence of DM is already starting to force $\delta_{B,k}$ to grow), there is also less smoothness for medium scales, that are not yet completely wiped out by Silk damping.

2.3 Conclusions

The objective of this work was one of pedagogical nature. It was not expected to find some new groundbreaking descriptions of the evolution of inhomogeneities in the Universe, but to be able to reproduce known behaviours of these inhomogeneities without the need of expensive and tasking numerical simulations. Using the wide range of approximations mentioned in this text we reached a set of solvable equations that we hoped would remain faithful to the actual evolution of δ_k. In general, this objective has been fulfilled. In the case of a purely baryonic Universe we hoped that the coupling of baryons and photons would mean that structures stop growing and start oscillating some time before their entry into the Horizon defined by their K_J and in fact we got this behaviour. By introducing photon diffusion, where the most important assumptions were taken, the fluctuations of smaller scales were damped and eventually wiped out, as was expected.

When we made the jump to a Universe with DM we recovered the results that $\delta_{DM,k}$ would stop growing when baryonic acoustic oscillations kicked in. Since there is no radiation-DM coupling, $\delta_{DM,k}$ kept growing, but at a much slower rate than before, as the dominant term for the gravitational potential, $\Omega_B\delta_{B,k} + \Omega_r\delta_{r,k}$, was stalling around 0.We expected $\delta_{DM,k}$ to present a small oscillation due to the mentioned dominant term in the gravitational potential oscillating, but this tranfered oscillation was perhaps too subtle. This oscillation is expected not only because previous results in literature, but because a qualitative analysis of the equation governing DM

$$\ddot{\delta}_{DM,k} + 2\frac{\dot{a}}{a}\ddot{\delta}_{DM,k} = \frac{3}{2}(\frac{\dot{a}}{a})^2(\Omega_B\delta_{B,k} + \Omega_{DM}\delta_{DM,k} + \Omega_r\delta_{r,k}) \tag{38}$$

and having obtained a oscillatory solution for $\delta_{B,k}, \delta_{r,k}$ which at some times will be negative, we should assume that some degree of oscillatory perturbations would translate into the evolution of DM structures. It is possible that the small size of these DM oscillations is a consequence of fails in the numerical treatment of the equations, althought it seems unlikely, given that the rest of wanted aspects of the solutions are obtained. A more extended analysis of both the numerical solution and the analytical process to reach the solved equations is needed to understand the source of this divergence from the known growth of DM. Taking into account photon diffusion does not solve this problem, but is satisfactory in the rest of aspects of the different solutions. The lightness of oscillations of DM means that its evolution won't be substantially different from the one result of adding radiation pressure, but we do observe a change in the evolution of $\delta_{B,k}$. Apart from suffering a process of Silk damping, as matter dominates the Universe the influence of the larger, still growing $\delta_{DM,k}$ makes $\delta_{B,k}$ start growing again. All the mentioned behaviours affect the shape of the power spectrum of each component. Its form has been already discussed, and it agrees with the observed sprectrums for values of k smaller than the one given by the Doppler effect (the main peak). Values of $P_{(k)}$ for k greater than where this peak is located are determined by the Sachs-Wolfe effect, not treated here.We can conclude that the equations solved are a good first approximation to linear evolution of small fluctuations for times between the end of inflation and the epoch of decoupling, and the approximations used to obtain them can be taken into account when performing an analytical treatment of this problem.

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