Dynamics of a monetary union: a stock-flow analysis

Luis Alberto Alonso and *Beatriz Guzmán*

Universidad Complutense de Madrid and *Universidad de La Laguna*

Luis Alberto Alonso. Departamento de Economía Aplicada III (Política Económica), Universidad Complutense de Madrid. Campus de Somosaguas 28223 - Pozuelo de Alarcón, Madrid. España. E-mail: alberto@ccee.ucm.es

Beatriz Guzmán (corresponding autor). Departamento de Dirección de Empresas e Historia Económica, Universidad de La Laguna. Campus de Guajara s/n. 38071 La Laguna, Tenerife. España. E-mail: bguzmanp@ull.es. Telephone number: (+34) 922 845 412

ABSTRACT

This paper models the dynamic path of a monetary union comprising two countries. The problems with this union initially arise when inflationary pressures from the member countries differ; the regulation provided by the instruments of common economic policy (common interest rate, extra-union exchange rate) is therefore inadequate for each country on its own. In this case, variations in price competitiveness produced in conjunction with the performance of the central bank ensure that inflation rates in both countries reach their target value. However, this result is achieved by generating permanent foreign deficits that fuel growth in the external debt stock, which in turn acts on the flows that generate it. The aim of this study is to analyse the stock-flow interaction that is unaddressed by the Theory of Optimum Currency Area. Based on the proposed model, criteria distinct from the usual criteria are derived to decide when it is appropriate to establish a monetary union.

Keywords: monetary union, stock-flow analysis, foreign debt, asymmetry, liquidity trap.

JEL codes: C62, C63, F45, E10, E12.

1 INTRODUCTION

This paper presents a model that can be used to determine the dynamic evolution and the final equilibrium of a monetary union formed by two blocs or countries. In the modelled union, a central bank establishes monetary policy by setting a common interest rate for the whole area so as to maintain a stable mean inflation rate. The two countries trade with each other and with the rest of the world in a regime of floating exchange rates.

This work can be seen as an attempt to overcome what could be considered a weakness in the Theory of Optimum Currency Area, weakness that in short we expose. According to the Theory of Optimum Currency Area (De Grauwe, 2016; Fleming, 1971; Kenen, 1969; Magnifico, 1973; McKinnon, 1963; Mundell, 1961), a country should be part of a monetary union when, as a result of a cost-benefit analysis, the microeconomic advantages of integration appear to be greater than the macroeconomic costs that integration entails. These costs must be measured by the difference between the values of the target variables that each of the member countries would reach inside and outside of the monetary union. In the monetary union, the target variables will be specific (values of income, inflation, foreign balances, and stocks of foreign debt of each of the member countries), and the instrument variables will be common (common interest rate and extra-union exchange rate). It therefore seems inevitable to build models that determine the results of the regulation exercised by these common instruments on the specific objectives mentioned.

This regulation is unsatisfactory when there are initial differences between countries or when countries receive asymmetric shocks because, in these cases, achievement of the union's objectives does not guarantee the objectives of each of the member countries. The single most important monetary problem arises when the partners have different propensities for inflation or when they receive exogenous shocks that unevenly affect inflation rates. These differences activate a mechanism that leads to variations in price competitiveness, resulting in deficits/surpluses that shrink/expand the economies of the member countries in such a way that their inflation rates equalize. However, permanent deficits/surpluses cause continued growth in the debt/assets of the debtor/creditor countries, and these stocks will act retroactively on the flows that generated them.

This dual stock-flow interaction is not addressed by the Theory of Optimum Currency Area. And without formalizing the process by which the external debt stock can stop growing and reach a situation that we would consider equilibrium and without determining its characteristics, the macroeconomic costs of integration and its appropriateness cannot be assessed. The model we develop here is an attempt to overcome this limitation.

The central thesis of this article is that initial differences in the values of certain characteristic parameters of monetary partners determine the relative creditor/debtor position of each country, the dynamic evolution of the external debt stock and whether a final balance that we describe as "steady state" is reached.

There are two types of initial differences. Some come from the demand side and manifest themselves in disparate types of natural interest, and others come from the supply side and are specified in various wage pressures in each country with respect to the country's average growth in productivity.

Variations in common interest rates and exchange rates and the mechanism of competitiveness, all of which were mentioned above, together equalize the inflation rates and make it for the value of target inflation. The process by which the above result is obtained is as follows. The negative trade balances of one of the blocs constitute a flow that feeds its external debt stock. Foreign debt grows; by contracting demand in the country, this growth improves its foreign balance. This produces a steady state in which the stock has stabilized and the debtor country obtains the trade surplus required to pay interest on the accumulated debt. The core of this model is, therefore, this dual stock-flow interaction: flow feeds a stock that is growing until said growth cancels out the flow. The proposed formulation fits into the current literature emphasizing the need to rigorously clarify and formulate interactions between flows and stocks (Caiani et al., 2004a, b; Caverzasi and Godin, 2015; Dos Santos, 2006; Godley and Lavoie, 2007, 2005, 2003; Taylor, 2008).

The behaviour of the modelled union depends crucially on what is referred to here as the degree of *asymmetry*. Asymmetry exists when foreign debt contracts the debtor economy more than the assets corresponding to that debt expand the creditor economy. When this occurs, as the debt of one country and the foreign assets of another grow, a contractionary result will be produced in the monetary union that will be offset by reductions in the interest rate by the central bank to maintain a stable mean rate of inflation.

The justification of the concept of asymmetry is based on the ideas of Koo (2008). It is assumed that foreign debt negatively affects aggregate demand in the debtor country because economic agents reduce their spending to offset the deterioration that foreign deficits cause in their balance sheets. Meanwhile, agents in the creditor country are not urged to correct the improvements in their balance sheets and do not increase their spending to the extent that debtors correct the imbalance in theirs.

The model provides three possibilities: that the steady state constancy of the foreign debt) is reached before the central bank reaches zero nominal interest rate; that the steady state occurs after the central bank achieves this nominal interest rate (in this case, the union will fall into the liquidity trap); and finally, that the steady state is never reached and the debt evolves explosively.

The proposed general model of the monetary union is too complex to be solved analytically. Dynamic results can only be obtained through simulation and will naturally not be general results. However, if it is assumed that the steady state is reached, for cases in which it is achieved the values of the endogenous variables can be analytically determined, and it is therefore possible to draw accurate conclusions about the final state of the monetary union.

In this work, two sub-models, the quantities sub-model and the prices sub-model, are formulated and solved. Each of these sub-models represents isolated mechanisms that act together in the general model. The quantities sub-model represents the influence of countries' incomes on their external balances and of their external balances on income. These interactions are called quantity reactions. The prices sub-model represents the influence of the relationship of price levels between countries in their current accounts and the effects of these balances on the inflation rate and the price competitiveness of each bloc. This mechanism is called price reactions. We could consider these sub-models as auxiliary models aimed at eliminating the opacity of the general model; that is, they provide the causal understanding that is made difficult by the complexity of the simulation model. This separate analysis is further justified if it is reasonable to assume that the quantity reactions occur before and more quickly than the price reactions but that when they act they neutralize and prevail over them.

It may be interesting to contrast to what extent the path and conclusions of the modelled monetary area account for the stylized facts of the most important monetary union at present, the European Monetary Union (EMU). In this regard, first, it is consistent with the model presented by Germany and adjacent countries, which have a weaker domestic aggregate demand and lower inflationary tensions than countries in the south and which form the creditor bloc in the EMU. It is consistent with the hypothesis of asymmetry that the EMU has a contractionary drift that keeps it in a recession despite the fact that the European Central Bank has reduced its nominal interest rate to zero. It is consistent with the relative speed that has been assumed for quantity reactions and price reactions that the EMU appears to be close to an equilibrium of quantities, i.e., that due to the economic contractions they are suffering, the southern countries are close to reaching surplus trade balances, while at the same time, it is generally accepted that the process of recovering price competitiveness has not yet been completed.

It is perhaps unavoidable to ask at what point of the path outlined by the model formulated here the EMU is presently found. To answer this, we must note that the EMU currently obtains an approximate foreign surplus of 3.5%, which the model does not include and that provisionally prevents the dynamics that the model predicts. The foreign surplus has reduced/eliminated the growth of foreign debt of the peripheral countries and the restorative effect of their competitiveness. Germany’s surplus increases not only due to the corresponding portion of the surplus of European and Monetary Union with the rest of the word but also because Germany's competitiveness against the peripheral countries has not been eroded. If the surplus with the rest of the world were to cease, the peripheral countries would again incur significant deficits, the growth of their foreign debt would resume, and the process of economic contraction and the reduction of inflation rates would continue. This means that when the external trade balance will be restored the EMU will have a long way to go to reach a steady state.

In the literature on the causes of the Eurozone crisis, there appears to be a largely shared view according to which the crisis can be attributed to the current account deficits in the peripheral countries in the prior period. However, there is disagreement over whether the deficits are the result of increased imports from these countries resulting from their higher growth rates (Comunale and Hessel, 2014; Storm and Naastepad, 2015) or rather the result of the loss of price competitiveness against the northern bloc. In turn, this reduction in price competitiveness can be attributed to autonomous wage pressures from the southern countries (Flassbeck and Spiecker, 2011) and the neo-mercantilist policies of Germany (Bibow, 2015) or, alternatively, that the highest rates of inflation they experienced by the countries in the south are the result of aggregate demand and economic activity that are more intense than those produced in the northern countries (Gaulier and Vicard, 2012).

Whatever relative weight of these explanations proves the most plausible, the proposed model could represent the actual process. Indeed, by modulating the values of the parameters of the functions that determine external balances, more weight may be given to the incomes of the two blocs or to the price relationships in determining the deficits of the debtor country. Additionally, we can find the importance of autonomous elevations of inflation rates by modifying the determinant parameters of what has been called 'pressure from the supply side'. Therefore, because it accounts for the various ways in which the Eurozone crisis could have occurred, it appears that the model is robust at this point.

The remainder of the article is organized as follows. In section II, the assumptions and equations of the general model are presented. In section III, the assumptions, results, and an explanation of the quantities sub-model and the prices sub-model are presented. In section IV, a graphical analysis of static equilibrium of the modelled union is shown. In section V, the results of the general model are set out in the form of 10 proposals, each of which is accompanied by graphics obtained from a simulation exercise as illustration. In section VI, the conclusions are presented.

2. THE GENERAL MODEL

We assume there is a monetary union formed by two countries or blocs called Spain and Germany; the superscripts S and G refer to each of these countries, respectively. It is recognized that these two countries are of the same size, which is reflected in their matching potential incomes, $Y^{n,S} = Y^{n,G} = Y^{n}$, and the currency they share is called the euro. The central bank of this union aims to ensure that the inflation rate in the area is 2% ($\bar{\pi}^{0} = 0.02$), and we assume that the bank has full credibility, so that the inflation rates expected by the economic actors in both blocs are 2%, i.e., $\pi_t^{e,S} = \pi_t^{e,G} = 0.02$.

An open model that includes the trade relations between the monetary partners and those that take place between each bloc and the rest of the world is formulated. We assume there is a single currency, the dollar, for the rest of the world and that the euro-dollar exchange rate fluctuates freely. It is recognized that there are no capital movements between the countries forming the monetary union and the rest of the world, so the exchange rate equilibrium is that at which the union's total current account balance is zero at all times.

We assume that the global inflation rate is constant and exogenous and equal to 2%, ($\pi^W = 0.02$) and that the global income level, Y^W, is also constant. Changes in the trade surpluses or deficits of each bloc of the union with the rest of the world will only depend on changes they experience with respect to the inflation rates and income levels of the monetary partners and on variations in the exchange rate.

2.1 Inflation rates

The inflation rate of each of the countries of the union is a weighted mean of the rate of domestic inflation, the inflation rate from imports of the monetary partner, and the rate attributable to imports from the rest of the world:

$$\pi_t^S = (1 - \lambda_1 - \lambda_2)\pi_t^{IS} + \lambda_1 \pi_t^G + \lambda_2 \pi^w \qquad (1)$$

$$\pi_t^G = (1 - \lambda_1 - \lambda_2)\pi_t^{IG} + \lambda_1 \pi_t^S + \lambda_2 \pi^w \qquad (2)$$

π_t^S, π_t^G are the inflation rates of the blocs, π_t^{IS}, π_t^{IG} are the domestic inflation rates, and λ_1, λ_2 are the relative weights of imports from the other country and from abroad in the production of each partner, respectively.

The domestic inflation rate of each member of the union is determined by using as a starting point an expression that indicates the general level of aggregate domestic prices, P_t^{IS}, P_t^{IG}. In the case of Spain, this expression is

$$P_t^{IS} = m\left(\frac{W_t^S}{T_t^{S}{}'}\right) \qquad (3)$$

where m is one more than *mark-up*, W_t^S indicates average monetary wages, and $T_t^{S\prime}$ is the mean labour productivity. Expressing (3) in terms of growth rates, we have

$$\pi_t^{IS} = \dot{m} + \dot{W}_t^S - \dot{T}^{S\prime} \tag{4}$$

A point above a variable denotes its growth rate. It is assumed that $\dot{m} = 0$ and that $\dot{W}_t^S$ depends on the growth of alleged real wages, $\dot{w}_t^{PS}$, as follows:

$$\dot{W}_t^S = \dot{w}_t^{PS} + \pi_t^{e,S} \tag{5}$$

It is considered that the union pressure exerted by workers, $\alpha^{S\prime}$, and the output gap of relative production, $\left(\frac{Y_t^S - Y^{n,S}}{Y^{n,S}}\right)$, are the factors that determine the percentage increase in alleged real wages, as shown in (6):

$$\dot{w}_t^{PS} = \alpha^{S\prime} + \beta^{S\prime}\left(\frac{Y_t^S - Y^{n,S}}{Y^{n,S}}\right), \tag{6}$$

in which $\beta^{S\prime}$ is a positive parameter.

Substituting (6) and (5) in (4) and considering $\pi_t^{e,S} = 0.02$, we have

$$\pi_t^{IS} = \left(\alpha^{S\prime} - \dot{T}^{S\prime}\right) + \beta^{S\prime}\left(\frac{Y_t^S - Y^{n,S}}{Y^{n,S}}\right) + 0.02 \tag{7}$$

Replacing π_t^{IS} in (1) with its value in (7) yields

$$\pi_t^S = (1-\lambda_1-\lambda_2)\left[\left(\alpha^{S\prime} - \dot{T}^{S\prime}\right) + \beta^{S\prime}\left(\frac{Y_t^S - Y^{n,S}}{Y^{n,S}}\right) + 0.02\right] + \lambda_1\pi_t^G + \lambda_2\pi^W \tag{8}$$

We can determine the rate of inflation in Germany by following a similar process to that discussed for the case of Spain:

$$\pi_t^G = (1-\lambda_1-\lambda_2)\left[\left(\alpha^{G\prime} - \dot{T}^{G\prime}\right) + \beta^{G\prime}\left(\frac{Y_t^G - Y^{n,G}}{Y^{n,G}}\right) + 0.02\right] + \lambda_1\pi_t^S + \lambda_2\pi^W \tag{9}$$

(8) and (9) are rewritten considering $\pi^W = 0.02$ and calling $\alpha^S = (1-\lambda_1-\lambda_2)\alpha^{S\prime}$, $\dot{T}^S = (1-\lambda_1-\lambda_2)\dot{T}^{S\prime}$, $\beta^S = (1-\lambda_1-\lambda_2)\beta^{S\prime}$, $\alpha^G = (1-\lambda_1-\lambda_2)\alpha^{G\prime}$, $\dot{T}^G = (1-\lambda_1-\lambda_2)\dot{T}^{G\prime}$, and $\beta^G = (1-\lambda_1-\lambda_2)\beta^{G\prime}$:

$$\pi_t^S = \left(\alpha^S - \dot{T}^S\right) + \beta^S\left(\frac{Y_t^S - Y^{n,S}}{Y^{n,S}}\right) + \lambda_1\pi_t^G + (1-\lambda_1)0.02 \tag{10}$$

$$\pi_t^G = \left(\alpha^G - \dot{T}^G\right) + \beta^G\left(\frac{Y_t^G - Y^{n,G}}{Y^{n,G}}\right) + \lambda_1\pi_t^S + (1-\lambda_1)0.02 \tag{11}$$

Considering $\beta^S = \beta^G = \beta$ and remembering that $Y^{n,S} = Y^{n,G} = Y^n$, the differences in inflation rate between the blocs of the union are expressed as

$$\left(\pi_t^S - \pi_t^G\right) = \left(\frac{1}{1+\lambda_1}\right)\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right) + \beta\left(\frac{Y_t^S - Y_t^G}{Y^n}\right)\right] \quad (12)$$

2.2 Foreign sectors of countries

Each country in the monetary union maintains trade relations with its monetary partner and with the rest of the world. The total trade balance of each bloc is obtained by adding its intra- and extra-union balances:

$$X_t^{TS} = -X_t^I + X_t^S \quad (13)$$

$$X_t^{TG} = X_t^I + X_t^G \quad (14)$$

where X_t^{TS}, X_t^{TG} is the total trade balance of each bloc, X_t^I is Germany's trade balance with Spain, and X_t^S and X_t^G represent the balance of each partner with the rest of the world. According to the model's assumptions, the total current account balance of the union for the equilibrium exchange rate is zero ($X_t^S + X_t^G = 0$); and because $X_t^S = -X_t^G$, we also see that $X_t^{TS} = -X_t^{TG}$.

The intra- and extra-union trade balances are made dependent on the intra- and extra-union ratio of incomes and prices, respectively, and the next expressions are proposed to determine these balances:

$$X_t^I = \delta_1 Ln\left[\left(\frac{Y_{t-1}^S}{Y_{t-1}^G}\right)^{j_1}\left(\frac{P_{t-1}^S}{P_{t-1}^G}\right)^{j_2}\right] \quad (15)$$

$$X_t^S = \delta_2 Ln\left[\left(v\frac{Y_{t-1}^W}{Y_{t-1}^S}\right)^{j_1}\left(\frac{P_{t-1}^W E_t}{P_{t-1}^S}\right)^{j_2}\right] \quad (16)$$

$$X_t^G = \delta_2 Ln\left[\left(v\frac{Y_{t-1}^W}{Y_{t-1}^G}\right)^{j_1}\left(\frac{P_{t-1}^W E_t}{P_{t-1}^G}\right)^{j_2}\right] \quad (17)$$

In (15), (16), and (17), δ_1, δ_2, j_1, j_2 are positive parameters, v is a coefficient used to adjust the size of the world economy to the size of each of the blocs of the monetary union, P_t^S, P_t^G, P_t^W are the general price levels, and E_t is the value of the euro-dollar exchange rate.

2.3 Foreign debt

Assuming that Spain is the debtor bloc, its foreign debt, D_t, has two origins. Part of its foreign debt results from its trade deficits with Germany, D_t^G; another part is due to deficits with the rest of the world, D_t^W:

$$D_t^G = \left(1 + r^{D,G}\right) D_{t-1}^G + X_t^I \tag{18}$$

$$D_t^W = \left(1 + r^{D,W}\right) D_{t-1}^W - X_t^S \tag{19}$$

$r^{D,G}$, $r^{D,W}$ are the real interest rates that each of the mentioned types of debt accrues. Adding (18) and (19) together, presuming $r^{D,G} = r^{D,W} = r^D$, and considering (13), we have:

$$D_t = \left(1 + r^D\right) D_{t-1} - X_t^{TS} \tag{20}$$

To simplify the formulation, it is assumed that Spain's debt to the rest of the world is financed by Germany, i.e., that Spain's excess of imports over exports with respect to the rest of the world is paid by selling Spain titles to Germany. It is also recognized that the interest accrued by these debts in each period is not paid but is continually refinanced. Therefore, the income of each bloc can be obtained without subtracting the accrued interest for Spain's debt in each period from the value of its output or adding them to Germany's output.

2.4 Extra-union exchange rate

The variation in the euro/dollar exchange rate is determined by

$$\dot{E}_t = \gamma\left(X_t^S + X_t^G\right) \tag{21}$$

where $\gamma < 0$.

2.5 Equilibrium output of the countries

The equilibrium output of the two countries is determined by an aggregate expenditure with four components. The first component is consumer demand depending on income during the period. A second component is the autonomous spending of the two blocs, A^S and A^G; it includes autonomous consumption, public spending, and autonomous investment. Third, we assume that part of spending depends negatively on the real interest rate r_t. The final component consists of the intra-union balance X_t^I and the extra-union balances X_t^S and X_t^G of the two blocs.

The last three components act on the output balance through a multiplier process; this process is assumed to be instantaneous and a multiplier coefficient of the unitary value. It is initially assumed that the rules of the monetary union require compliance with the budgetary balance. At the end of this work, this assumption is removed to permit study of the effects of the use of fiscal policy, which will be manifested in changes in the autonomous spending component A^S, A^G.

As stated in the Introduction, the concept of asymmetry contemplated in the model determines that the foreign debt and foreign assets have a negative or positive effect through parameters ε^S and ε^G respectively, on equilibrium expenditure and production in each of the blocs.

According to the above, the equations for equilibrium income in Spain and Germany are

$$Y_t^S = A^S - \varepsilon^S D_t - B^S r_t - X_t^I + X_t^S \tag{22}$$

and

$$Y_t^G = A^G + \varepsilon^G D_t - B^G r_t + X_t^I + X_t^G \tag{23}$$

where B^S and B^G are positive parameters.

2.6 Central bank and interest rates

Because coinciding potential income levels have been assumed, the inflation of the entire area, $\overline{\pi}_t$, is the simple arithmetic mean of the inflation rates of the blocs that compose it:

$$\overline{\pi}_t = \frac{\pi_t^S + \pi_t^G}{2} \tag{24}$$

The real interest rate r_t is obtained by subtracting the inflation rate expected by economic agents from the nominal interest rate i_t, i.e., $r_t = i_t - 0.02$. The minimum interest rate r^{min} is defined as the difference between the zero nominal interest rate and the expected inflation rate, i.e., $r^{min} = 0 - 0.02 = -0.02$.

The real interest rate is the instrument that the central bank manages in an attempt to eliminate or reduce any deviations between the inflation rate and the target rate, $\overline{\pi}^0 = 2\%$. Considering (24), the direct relationship between the inflation rate of each bloc and the relative output gap (equations (10) and (11)) and the inverse relationship between output and the real interest rate (equations (22) and (23)), it is assumed that the monetary authority acts in accordance with the following rule of monetary policy:

$$r_t = r_{t-1} + \rho\left(\bar{\pi}_{t-1} - \bar{\pi}^0\right) \text{ if } r_t > r^{min} \quad (25)$$

$$r_t = r^{min} \text{ if } r_t \leq r^{min} \quad (25LT)$$

where ρ is a positive parameter that reflects the sensitivity of the real interest rate to the deviation between inflation of the monetary union and the target inflation rate.

The interest rate r_t in expression (25) is established by the central bank, which adjusts it to an activist rule through a process of trial and error to bring the union's mean inflation rate to 2%. In addition, $\hat{r}_t$ is defined as the interest rate in period t through which the central bank would achieve this objective.

2.7 Potential output of the countries

The potential output is defined as the level of output that allows a bloc to achieve an inflation rate of 2% when the following conditions exist: (a) the difference between the wage pressure of workers and increased mean work productivity is zero; and (b) the effect of imported inflation from the other bloc is zero, i.e., $\lambda_1 = 0$- o, where $\lambda_1 \neq 0$ and the inflation of the other bloc is 2%. Therefore, although a bloc may have an inflation rate of 2%, its output will not equal the potential output when there are existing tensions in supply or when the imported inflation rate of the other bloc is other than 2%.

2.8 Natural interest rates of countries

It is considered that the natural interest rate of a bloc is the rate that, when the foreign debt and the total foreign balance are zero ($D_t = 0$, $X_t^I = X_t^S = X_t^G = 0$), allows to reach the potential income of the bloc. Using (22) and (23) to formulate the natural interest rates of both countries, we have

$$r^{n,S} = \frac{A^S - Y^n}{B^S} \quad (26)$$

$$r^{n,G} = \frac{A^G - Y^n}{B^G} \quad (27)$$

2.9 Liquidity trap

The liquidity trap is defined as a situation of the monetary union in which the real interest rate is the minimum ($r_t = r^{min}$) and the mean inflation rate of the union is lower than 2% ($\bar{\pi}_t < 0.02$). This situation appears in this model when the nominal interest rate is zero ($i_t = 0$),because the expected inflation rate is assumed to be always 2%.

3. QUANTITY AND PRICE SUB-MODELS: ASSUMPTIONS, RESOLUTION AND EXPLANATION

In this section, we present, in two different sub-sections, the simplifications made in the general model to obtain the quantities and prices sub-models, and each is resolved separately in two cases, when the central bank has leeway to vary the interest rate and in the liquidity trap regime. In the prices sub-model, we also explore the particular case in which, with the minimum interest rate, the inflation rates of the blocs have different sensitivities to relative output gaps. We also provide an explanation of each of the sub-models.

3.1 Quantities sub-model

3.1.1 Assumptions

The simplifications made to obtain the quantities sub-model from the general model are as follows: we make $\delta_2 = 0$ in (16) and (17) and $j_2 = 0$ in (15); we therefore have $X_t^S = 0$ and $X_t^G = 0$, and equations (22) and (23) become

$$Y_t^S = A^S - \varepsilon^S D_t - B^S r_t - X_t^I \qquad (22Q)$$

$$Y_t^G = A^G + \varepsilon^G D_t - B^G r_t + X_t^I. \qquad (23Q)$$

It is assumed that $\varepsilon^S \geq \varepsilon^G$ and $A^S > A^G$.

When $j_2 = 0$,

$$X_t^I = \delta_1 Ln\left(\frac{Y_{t-1}^S}{Y_{t-1}^G}\right)^{j_1}. \qquad (15Q)$$

That is, the intra-union trade balance in this sub-model is determined solely by the relationship between the income levels of the two blocs.

The equation of foreign debt in which D_t is only intra-union debt would be

$$D_t = \left(1 + r^D\right)D_{t-1} + X_t^I \qquad (20Q)$$

In (10) and (11), we make $\beta^S = \beta^G = 0$ y $\left(\alpha^S - \dot{T}^S\right) = \left(\alpha^G - \dot{T}^G\right) = 0$. Both assumptions imply that $\pi_t^S = \pi_t^G = 0.02$ for every *t*.

The central bank aims to achieve the potential income of the union, $\overline{Y}^n$, and regulates the interest rate in an attempt to eliminate or reduce any differences in the income of the whole area, $\overline{Y}_t$; thus,

$$r_t = r_{t-1} + \chi\left(\overline{Y}_{t-1} - \overline{Y}^n\right) \text{ if } r_t > r^{min} \quad (25Q)$$

$$r_t = r^{min} \text{ if } r_t \le r^{min} \quad (25QLT)$$

3.1.2 Resolution

R_t is defined as the relationship between the incomes of the countries in the monetary union in period t, as follows:

$$R_t \equiv \frac{Y_t^S}{Y_t^G}. \quad (28)$$

Using (28) in (15Q), we have

$$X_t^I = \delta_1 Ln(R_{t-1})^{j_1}. \quad (29)$$

Substituting in (22Q) and (23Q) X_t^I for its value in (29), we have

$$Y_t^S = A^S - \varepsilon^S D_t - B^S r_t - \delta_1 Ln\left(R_{t-1}\right)^{j_1} \quad (30)$$

$$Y_t^G = A^G + \varepsilon^G D_t - B^G r_t + \delta_1 Ln\left(R_{t-1}\right)^{j_1} \quad (31)$$

Dividing (30) by (31), assuming $B^S = B^G = B$:

$$R_t = \frac{A^S - \varepsilon^S D_t - Br_t - \delta_1 Ln\left(R_{t-1}\right)^{j_1}}{A^G + \varepsilon^G D_t - Br_t + \delta_1 Ln\left(R_{t-1}\right)^{j_1}} \quad (32)$$

The central bank's objective is to make the currency area reach its potential income; that is,

$$2Y^n = Y_t^S + Y_t^G \quad (33)$$

At the outset, when the union is formed $D_0 = 0$ and $X_0^I = 0$, the target interest rate that the central bank should establish to ensure compliance with (33) can be found from

$$2Y^n = A^S + A^G - 2B\,\hat{r}_0. \quad (34)$$

Solving for $\hat{r}_0$ from (34), we have

$$\hat{r}_0 = \frac{A^S + A^G - 2Y^n}{2B} \tag{35}$$

Using (26) and (27) in (35) to express $\hat{r}_0$ in terms of the natural interest rates of the blocs of the union yields

$$\hat{r}_0 = \frac{1}{2}\left(r^{n,S} + r^{n,G}\right). \tag{36}$$

According to (36), when the monetary union is formed, the central bank assumes that the sum of the incomes of the two countries is equal to twice the potential income of both, establishing an interest rate that is the simple arithmetic mean of the natural interest rates of the blocs.

At any moment *t*, the interest rate to be established by the monetary authority to reach the potential income of the union, $\hat{r}_t$, is obtained by replacing Y_t^S and Y_t^G in (33) with their values in (30) and (31), respectively, as shown in (37):

$$2Y^n = A^S + A^G - \left(\varepsilon^S - \varepsilon^G\right)D_t - 2B\,\hat{r}_t \tag{37}$$

It is assumed that the value of χ in (25Q) ensures that the interest rate that the central bank sets is always its target interest rate $\hat{r}_t$

Substracting (37) for (34), we have

$$\hat{r}_t = \hat{r}_0 - \frac{\left(\varepsilon^S - \varepsilon^G\right)}{2B}D_t \tag{38}$$

$\hat{r}_t$ can be expressed in terms of the natural interest rates, substituting $\hat{r}_0$ in (38) by its value in (36):

$$\hat{r}_t = \frac{1}{2}\left(r^{n,S} + r^{n,G}\right) - \frac{\left(\varepsilon^S - \varepsilon^G\right)}{2B}D_t \tag{39}$$

The ratio of the blocs' income resulting from the performance of the central bank in its attempt to match the union's income to its potential income is obtained by replacing r_t in (32) with $\hat{r}_t$ obtained in (38):

$$R_t = \frac{A^S - \left(\frac{\varepsilon^S + \varepsilon^G}{2}\right)D_t - B\,\hat{r}_0 - \delta_1 Ln\left(R_{t-1}\right)^{j_1}}{A^G + \left(\frac{\varepsilon^S + \varepsilon^G}{2}\right)D_t - B\,\hat{r}_0 + \delta_1 Ln\left(R_{t-1}\right)^{j_1}} \tag{40}$$

We can expect that in its convergence towards long-term equilibrium, the time trajectory from (40) is oscillating. Effectively, $R_t = R_{t-1} = R^*$. If $R_{t-1} < R^*$, equation (40) shows that $R_t > R^*$, and $R_t > R_{t-1}$. In addition, if $R_t > R^*$, then $R_{t+1} < R^*$ and $R_{t+1} < R_t$.

To account for the effect of foreign debt in (40), X_t^I in (20Q) should be replaced with its value in (29):

$$D_t = D_{t-1}\left(1 + r^D\right) + \delta_1 Ln(R_{t-1})^{j_1} \qquad (41)$$

Equations (40) and (41) form a system of two nonlinear difference equations whose variables are R_t and D_t. These equations describe the dynamic path of the union being modelled.

The value of equilibrium of debt, D^*, is obtained by assuming a constant value for it in (41):

$$D^* = -\frac{\delta_1}{r^D} Ln(R^*)^{j_1} \qquad (42)$$

The ratio of equilibrium incomes, if solvable, can be obtained by replacing D^* in (40) with its value in (42):

$$R^* = \frac{A^S - B\hat{r}_0 - \delta_1 Ln\left(R^*\right)^{j_1}\left[1 - \frac{\left(\varepsilon^S + \varepsilon^G\right)}{2r^D}\right]}{A^G - B\hat{r}_0 + \delta_1 Ln\left(R^*\right)^{j_1}\left[1 - \frac{\left(\varepsilon^S + \varepsilon^G\right)}{2r^D}\right]}. \qquad (43)$$

When the monetary union falls into the liquidity trap, the income ratio is

$$R_t = \frac{A^S - \varepsilon^S D_t - Br^{\min} - \delta_1 Ln\left(R_{t-1}\right)^{j_1}}{A^G + \varepsilon^G D_t - Br^{\min} + \delta_1 Ln\left(R_{t-1}\right)^{j_1}} \qquad (44)$$

The dynamic trajectory of the union modelled in this regime is given by equations (44) and (41), which form a system of two nonlinear difference equations whose variables are R_t and D_t. D_t in (44) is the first value for which the interest rate that the central bank should establish is less than the minimum interest rate, i.e., it fulfils

$$\hat{r}_0 - \frac{\left(\varepsilon^S - \varepsilon^G\right)}{2B} D_t < r^{min} \qquad (45)$$

The expression of the ratio of equilibrium incomes, if equilibrium is reached in this regime, is obtained by replacing D^* in (44) with its value in (42):

$$R^{*} = \frac{A^{S} - Br^{\min} - \left(1 - \frac{\varepsilon^{S}}{r^{D}}\right)\delta_{1} Ln\left(R^{*}\right)^{j_{1}}}{A^{G} - Br^{\min} + \left(1 - \frac{\varepsilon^{G}}{r^{D}}\right)\delta_{1} Ln\left(R^{*}\right)^{j_{1}}} \qquad (46)$$

<u>*3.1.3 Explanation*</u>

In short, we have assumed a monetary union consisting of two blocs that have potential coinciding incomes and different natural interest rates at the beginning of the monetary integration process. The central bank's goal is to ensure that the whole area reaches its potential income; to this end, it sets an interest rate that is the simple arithmetic mean of the natural interest rates of the member countries (expression (36)). Although adequate for the union as a whole, this interest rate is not adequate for each bloc separately because it is excessive for the country with the lower natural rate and too low for the other country, giving the former an income lower than its potential income and the latter an income higher than its potential income.

The countries in the monetary area trade with each other, and the imports of each directly depend on their level of income (expression (15Q)). Under the specified conditions, the bloc with the higher natural interest rate would have a trade deficit, and its partner would have a symmetrical surplus. The deficit or surplus would reduce or increase, respectively, the income of each bloc, thereby lessening the corresponding deficit or surplus in the following period. Through an oscillatory process, this would lead to values of the foreign balance justified by a given relationship between incomes that, at the same time, determines this relationship (expression (40)).

The deficit foreign balance of the country with the higher natural interest rate will increase the external debt stock (equation (41)). If there is no asymmetry, the equilibrium incomes of the two blocs would vary by the same amount in opposite directions, the total income of the union would not move away from the potential income, and the central bank would not need to modify the initial interest rate (expression (36)). This process would end when the debt stops growing, which requires that the initial ratio of the countries' incomes be reversed. Specifically, this occurs when the country with the higher initial income obtains a lower relative income with which it can reach a coinciding trade surplus with the interests of the accumulated debt. At this time, if certain dynamic conditions are met, the final equilibrium would be reached (equations (42) and (43)).

In the event that there is asymmetry, the total income of the union would decline and would remain below potential, and the central bank would have to reduce the interest rate to restore the desired equilibrium (expression (38)). This reduction would increase Germany's income with respect to its previous value but would insufficiently offset Spain's contraction caused by its indebtedness. The process of increasing income in Germany and the more than proportional reduction of Spain's income would reverse the relationship between the incomes of the two countries, and the same situation would be reached as in the previous case - a value of trade surplus in Spain equal to the interest of the previous debt. It could happen that before reaching this equilibrium the central bank's chances of reducing the interest rate are exhausted because the rate reaches the minimum threshold (equation (45)). In this situation, it would be more difficult for the indebted country (Spain) to achieve a trade surplus that guarantees the constancy of the debt, not only because Germany's income would not grow due to the central bank's actions but also because Germany’s income would be reduced by the contraction in her exports resulting from Spain's reduction in income. Therefore, the income difference in favour of Germany that is needed to generate the surplus in Spain would be more difficult to achieve or could not be reached (equations (46) and (42)).

3.2 Prices sub-model

3.2.1 Assumptions

The simplifications that define the prices sub-model are as follows. Intra- and extra-union trade balances are determined only by the relative price levels; this implies making $j_1 = 0$ in (15), (16) and (17):

$$X_t^I = \delta_1 Ln\left[\left(\frac{P_{t-1}^S}{P_{t-1}^G}\right)^{j_2}\right] \tag{15P}$$

$$X_t^S = \delta_2 Ln\left[\left(\frac{P_{t-1}^W E_t}{P_{t-1}^S}\right)^{j_2}\right] \tag{16P}$$

$$X_t^G = \delta_2 Ln\left[\left(\frac{P_{t-1}^W E_t}{P_{t-1}^G}\right)^{j_2}\right] \quad (17P)$$

To operate this more easily, it is assumed that the determination of the exchange rate provided by equation (21) is made through an expression obtained assuming that the theory of purchasing power parity is fulfilled, i.e.,

$$\frac{\overline{P}_{t-1}}{P_{t-1}^W E_t} = 1 \quad (47)$$

where $\overline{P}_t$ is the general mean price level of the union. Expressing (47) in terms of growth rates, we obtain the equation of exchange rate variation, which is

$$\dot{E}_t = \frac{\pi_{t-1}^S + \pi_{t-1}^G}{2} - \pi_{t-1}^W \quad (21P)$$

Formulating (15P), (16P) and (17P) in growth rates, we have

$$X_t^I = X_{t-1}^I + \delta_1 j_2 \left(\pi_{t-1}^S - \pi_{t-1}^G\right) \quad (48)$$

$$X_t^S = X_{t-1}^S + \delta_2 j_2 \left(\pi_{t-1}^W - \pi_{t-1}^S + \dot{E}_t\right) \quad (49)$$

$$X_t^G = X_{t-1}^G + \delta_2 j_2 \left(\pi_{t-1}^W - \pi_{t-1}^G + \dot{E}_t\right) \quad (50)$$

Replacing $\dot{E}_t$ in (49) and (50) with its value from (21P) yields

$$X_t^S = X_{t-1}^S + \frac{\delta_2 j_2}{2}\left(\pi_{t-1}^G - \pi_{t-1}^S\right) \quad (51)$$

$$X_t^G = X_{t-1}^G + \frac{\delta_2 j_2}{2}\left(\pi_{t-1}^S - \pi_{t-1}^G\right) \quad (52)$$

Spain's total balance is obtained by substituting X_t^I and X_t^S in (13) with their values in (48) and (51), respectively; analogously, for the total balance of Germany, X_t^I and X_t^G (14) are replaced with their values in (48) and (52), as follows:

$$X_t^{TS} = X_{t-1}^{TS} + \delta' \left(\pi_{t-1}^G - \pi_{t-1}^S\right) \quad (53)$$

$$X_t^{TG} = X_{t-1}^{TG} + \delta' \left(\pi_{t-1}^S - \pi_{t-1}^G\right) \quad (54)$$

where $\delta' = j_2\left(\delta_1 + \frac{\delta_2}{2}\right)$.

The other equations in this sub-model are those set forth for the general model.

<u>*3.2.2 Resolution*</u>

Universidad de La Laguna
Oficina de Sede Electrónica
Entrada
Nº registro: 2017/54291
Nº reg. oficina: OF002/2017/43067
Fecha: 11/11/2017 09:41:40

Replacing Y_t^S, Y_t^G in (12) with expressions (22) and (23), respectively, assuming $B^S = B^G = B$ and considering $X_t^S = -X_t^G$ yields

$$\left(\pi_t^S - \pi_t^G\right) = \frac{1}{(1+\lambda_1)}\left\{\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right] + \frac{\beta}{Y^n}\left[\left(A^S - A^G\right) - \left(\varepsilon^S + \varepsilon^G\right)D_t - 2X_t^{TG}\right]\right\} \tag{55}$$

(55) is lagged a period, and the value obtained is used in (54) to replace $\left(\pi_{t-1}^S - \pi_{t-1}^G\right)$:

$$\begin{aligned} X_t^{TG} &= \left[1 - \frac{2\delta'\beta}{(1+\lambda_1)Y^n}\right]X_{t-1}^{TG} - \frac{\delta'\beta\left(\varepsilon^S + \varepsilon^G\right)}{(1+\lambda_1)Y^n}D_{t-1} + \frac{\delta'\beta}{(1+\lambda_1)Y^n}\left(A^S - A^G\right) + \\ &+ \frac{\delta'}{(1+\lambda_1)}\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right] \end{aligned} \tag{56}$$

Solving for X_{t-1}^{TG} from the expression that offsets (20) a period and remembering that $X_t^{TS} = -X_t^{TG}$, we have

$$X_{t-1}^{TG} = D_{t-1} - \left(1 + r^D\right)D_{t-2} \tag{57}$$

X_{t-1}^{TG} is replaced in (56) by its value in (57):

$$\begin{aligned} X_t^{TG} &= \left\{1 - \frac{\delta\beta\left[2 + \left(\varepsilon^S + \varepsilon^G\right)\right]}{(1+\lambda_1)Y^n}\right\}D_{t-1} - \left\{\left[1 - \frac{2\delta\beta}{(1+\lambda_1)Y^n}\right]\left(1 + r^D\right)\right\}D_{t-2} + \\ &+ \frac{\delta\beta}{(1+\lambda_1)Y^n}\left(A^S - A^G\right) + \frac{\delta}{(1+\lambda_1)}\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right] \end{aligned} \tag{58}$$

The equation of the temporal trajectory of foreign debt is obtained by replacing X_t^{TG} in (20) with its value in (58), remembering that $X_t^{TS} = -X_t^{TG}$:

$$\begin{aligned} D_t &= \left\{2 + r^D - \frac{\delta'\beta\left[2 + \left(\varepsilon^S + \varepsilon^G\right)\right]}{(1+\lambda_1)Y^n}\right\}D_{t-1} - \left\{\left[1 - \frac{2\delta'\beta}{(1+\lambda_1)Y^n}\right]\left(1 + r^D\right)\right\}D_{t-2} + \\ &+ \frac{\delta'\beta}{(1+\lambda_1)Y^n}\left(A^S - A^G\right) + \frac{\delta'}{(1+\lambda_1)}\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right] \end{aligned} \tag{59}$$

The value of the long-term equilibrium of foreign debt is determined[1] by testing (59) with a constant solution for D_t such that

$$D^* = \frac{\left(A^S - A^G\right) + \dfrac{Y^n}{\beta}\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right]}{\left(\varepsilon^S + \varepsilon^G\right) - 2r^D} \tag{60}$$

Using (26) and (27), (60) is expressed in terms of the natural interest rates of the blocs, as follows:

$$D^* = \frac{B\left(r^{n,S} - r^{n,G}\right) + \frac{Y^n}{\beta}\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right]}{\left(\varepsilon^S + \varepsilon^G\right) - 2r^D} \quad (61)$$

It is observed that the long-term equilibrium value of foreign debt given in (61) depends on the difference between the natural interest rates of the monetary partners $r^{n,S}$ and $r^{n,G}$, the divergence between their supply tensions $\left(\alpha^G - \dot{T}_t^G\right)$ and $\left(\alpha^S - \dot{T}_t^S\right)$, the effect exerted by foreign debt on aggregate demand for each partner (ε^S, ε^G), and the interest rate r^D earned on debt.

To analyse the stability of the long-term equilibrium of foreign debt (expression (61)), it is necessary to obtain the value of the characteristic roots b_1 and b_2 of the equation reduced from (59):

$$b_1, b_2 = \frac{\left\{2 + r^D - \frac{\delta' \beta\left[2 + \left(\varepsilon^S + \varepsilon^G\right)\right]}{\left(1+\lambda_1\right)Y^n}\right\}}{2} \pm \\ \pm \frac{\sqrt{\left\{2 + r^D - \frac{\delta'' \beta\left[2 + \left(\varepsilon^S + \varepsilon^G\right)\right]}{\left(1+\lambda_1\right)Y^n}\right\}^2 - 4\left[1 - \frac{2\delta' \beta}{\left(1+\lambda_1\right)Y^n}\right]\left(1 + r^D\right)}}{2} \quad (62)$$

Given the complexity of solving (62), we worked with the inequalities proposed by Gandolfo (1976: 58) that constitute a set of necessary, sufficient conditions that ensure that the characteristic roots b_1 and b_2, whether real or complex, are less than one in absolute value:

$$1^{st}: \; r^D < \frac{\left(\varepsilon^S + \varepsilon^G\right)}{2}$$

$$2^{nd}: \; r^D < \frac{2\delta' \beta}{\left(1+\lambda_1\right)Y^n - 2\delta' \beta} \quad (63)$$

$$3^{rd}: \; r^D < \frac{4\left(1+\lambda_1\right)Y^n - \delta' \beta\left[4 + \left(\varepsilon^S + \varepsilon^G\right)\right]}{2\left[\delta' \beta - \left(1+\lambda_1\right)Y^n\right]}$$

Verification of the requirements in (63) ensures that the long-term equilibrium value of debt (61) is stable.

To obtain the interest rate $\hat{r}_t$, which in period *t* enables the inflation rate of area $\bar{\pi}_t$ to coincide with the target inflation rate ($\bar{\pi}_t = \bar{\pi}^0 = 0.02$), (24) is reformulated as

$$\left(\pi_t^S + \pi_t^G\right) = 0.04 \quad (64)$$

(10) and (11) are added together, assuming $\beta^S = \beta^G = \beta$:

$$\left(\pi_t^S+\pi_t^G\right)=\frac{\beta}{(1-\lambda_1)Y^n}\left(Y_t^S+Y_t^G\right)-2\frac{\beta}{(1-\lambda_1)}+0.04+\frac{1}{(1-\lambda_1)}\left[\left(\alpha^S-\dot{T}^S\right)+\left(\alpha^G-\dot{T}^G\right)\right] \quad (65)$$

Replacing $\left(\pi_t^S+\pi_t^G\right)$ in (64) with its value in (65), we obtain

$$\frac{1}{(1-\lambda_1)}\left(\hat{Y}_t^S+\hat{Y}_t^G\right)-2\frac{Y^n}{(1-\lambda_1)}+\frac{Y^n}{(1-\lambda_1)}\frac{\left[\left(\alpha^S-\dot{T}^S\right)+\left(\alpha^G-\dot{T}^G\right)\right]}{\beta}=0 \quad (66)$$

where $\hat{Y}_t^S$ and $\hat{Y}_t^G$ are the incomes that satisfy (64).

Adding (22) and (23) and assuming that $B^S=B^G=B$, according to the definition of $\hat{r}_t$, yields

$$\left(\hat{Y}_t^S+\hat{Y}_t^G\right)=\left(A^S+A^G\right)-\left(\varepsilon^S-\varepsilon^G\right)D_t-2B\hat{r}_t. \quad (67)$$

Substituting $\left(\hat{Y}_t^S+\hat{Y}_t^G\right)$ in (66) with its value in (67) and using (26) and (27), we obtain the central bank's target interest rate $\hat{r}_t$ for any value of debt D_t:

$$\hat{r}_t=\frac{1}{2}\left(r^{n,S}+r^{n,G}\right)-\frac{\left(\varepsilon^S-\varepsilon^G\right)}{2B}D_t+\frac{Y^n}{2B}\frac{\left[\left(\alpha^S-\dot{T}^S\right)+\left(\alpha^G-\dot{T}^G\right)\right]}{\beta} \quad (68)$$

At the moment the union is formed, when debt is zero, the central bank's target interest rate, $\hat{r}_0$, is

$$\hat{r}_0=\frac{1}{2}\left(r^{n,S}+r^{n,G}\right)+\frac{Y^n}{2B}\frac{\left[\left(\alpha^S-\dot{T}^S\right)+\left(\alpha^G-\dot{T}^G\right)\right]}{\beta}. \quad (69)$$

$\hat{r}_t$ can be expressed in (68) in terms of $\hat{r}_0$ using the definition of the latter obtained in (69), as follows:

$$\hat{r}_t=\hat{r}_0-\frac{\left(\varepsilon^S-\varepsilon^G\right)}{2B}D_t \quad (70)$$

To determine the central bank's target interest rate in the final equilibrium, $\hat{r}^*$, we replace D_t in (68) with its value in (61):

$$\hat{r}^{*} = \frac{Y^n}{2B\beta}\left\{\left[\left(\alpha^S - \dot{T}^S\right) + \left(\alpha^G - \dot{T}^G\right)\right] - \left\{\frac{\left(\varepsilon^S - \varepsilon^G\right)\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right]}{\left(\varepsilon^S + \varepsilon^G\right) - 2r^D}\right\}\right\} + \frac{1}{2} r^{n,G}\left[1 + \frac{\left(\varepsilon^S - \varepsilon^G\right)}{\left(\varepsilon^S + \varepsilon^G\right) - 2r^D}\right] + \frac{1}{2} r^{n,S}\left[1 - \frac{\left(\varepsilon^S - \varepsilon^G\right)}{\left(\varepsilon^S + \varepsilon^G\right) - 2r^D}\right] \quad (71)$$

The sufficient condition under which the economy does not fall into the liquidity trap is, obviously, $\hat{r}^{*} > r^{\min}$, which is expressed as

$$\frac{Y^n}{2B\beta}\left\{\left[\left(\alpha^S - \dot{T}^S\right) + \left(\alpha^G - \dot{T}^G\right)\right] - \left\{\frac{\left(\varepsilon^S - \varepsilon^G\right)\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right]}{\left(\varepsilon^S + \varepsilon^G\right) - 2r^D}\right\}\right\} + \frac{1}{2} r^{n,G}\left[1 + \frac{\left(\varepsilon^S - \varepsilon^G\right)}{\left(\varepsilon^S + \varepsilon^G\right) - 2r^D}\right] + \frac{1}{2} r^{n,S}\left[1 - \frac{\left(\varepsilon^S - \varepsilon^G\right)}{\left(\varepsilon^S + \varepsilon^G\right) - 2r^D}\right] > r^{min} \quad (72)$$

To obtain the value of Spain's total foreign trade balance when foreign debt stabilizes at its equilibrium value, we make $D_t = D_{t-1} = D^*$ in (20):

$$X^{TS*} = r^D D^* \quad (73)$$

We can determine the value of the blocs' equilibrium incomes $\hat{Y}^{S*}$ and $\hat{Y}^{G*}$ considering that the values of the independent variables from (22) and (23) are those from equilibrium ($D_t = D^*$, $r_t = \hat{r}^{*}$, $X_t^{TS} = X^{TS*}$) and are given by (61), (71) and (73), respectively, as follows:

$$\hat{Y}^{S*} = Y^n - \frac{Y^n}{\beta}\left(\alpha^S - \dot{T}^S\right) \quad (74)$$

$$\hat{Y}^{G*} = Y^n - \frac{Y^n}{\beta}\left(\alpha^G - \dot{T}^G\right) \quad (75)$$

3.2.3 Liquidity trap and different betas

It is interesting to analyse the implications of the case in which the blocs' inflation rates have different sensitivities to output gaps in a context in which the central bank has no leeway to vary the interest rate. For $r_t = r^{\min}$, the blocs' incomes are expressed as

$$Y_t^S = A^S - \varepsilon^S D_t - Br^{min} - X_t^{TG} \quad (76)$$

$$Y_t^G = A^G + \varepsilon^G D_t - Br^{min} + X_t^{TG} \tag{77}$$

(12) is rewritten when $\beta^G \neq \beta^S$:

$$\left(\pi_t^S - \pi_t^G\right) = \frac{1}{(1+\lambda_1)}\left\{\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right] + \beta^S\left(\frac{Y_t^S - Y^n}{Y^n}\right) - \beta^G\left(\frac{Y_t^G - Y^n}{Y^n}\right)\right\} \tag{12LT}$$

Y_t^S, Y_t^G are replaced in (12LT) by their values in (76) and (77), respectively, as follows:

$$\left(\pi_t^S - \pi_t^G\right) = \frac{1}{(1+\lambda_1)}\left\{\begin{array}{l}\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right] + \\ + \dfrac{1}{Y^n}\left[\begin{array}{l}\left(\beta^S A^S - \beta^G A^G\right) - \left(\beta^S \varepsilon^S + \beta^G \varepsilon^G\right) D_t - \\ - \left(\beta^S - \beta^G\right) Br^{min} - \left(\beta^S + \beta^G\right) X_t^{TG} - \left(\beta^S - \beta^G\right) Y^n\end{array}\right]\end{array}\right\} \tag{78}$$

(78) is offset a period, and the value obtained is used in (54) to replace $\left(\pi_{t-1}^S - \pi_{t-1}^G\right)$:

$$X_t^{TG} = \left[1 - \frac{\delta'\left(\beta^S + \beta^G\right)}{(1+\lambda_1)Y^n}\right] X_{t-1}^{TG} + \frac{\delta'}{(1+\lambda_1)}\left\{\begin{array}{l}\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right] + \\ + \dfrac{1}{Y^n}\left[\begin{array}{l}\left(\beta^S A^S - \beta^G A^G\right) - \left(\beta^S \varepsilon^S + \beta^G \varepsilon^G\right) D_{t-1} - \\ - \left(\beta^S - \beta^G\right) Br^{\min} - \left(\beta^S - \beta^G\right) Y^n\end{array}\right]\end{array}\right\} \tag{79}$$

Replacing X_{t-1}^{TG} in (79) by its value in (57), we have

$$\begin{array}{l} X_t^{TG} = \left[1 - \dfrac{\delta'\left(\beta^S + \beta^G\right)}{(1+\lambda_1)Y^n}\right]\left[D_{t-1} - D_{t-2}\left(1 + r^D\right)\right] + \dfrac{\delta'}{(1+\lambda_1)Y^n}\left(\beta^S \varepsilon^S + \beta^G \varepsilon^G\right) D_{t-1} + \\ \dfrac{\delta'}{(1+\lambda_1)Y^n}\left[\beta^S\left(A^S - Br^{\min} - Y^n\right) - \beta^G\left(A^G - Br^{\min} - Y^n\right)\right] + \dfrac{\delta'}{(1+\lambda_1)}\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right] \end{array} \tag{80}$$

The temporal trajectory of foreign debt is obtained by replacing X_t^{TG} in (20) with its value in (80), remembering that $X_t^{TS} = -X_t^{TG}$:

$$\begin{array}{l} D_t = \left\{2 + r^D - \dfrac{\delta'}{(1+\lambda_1)Y^n}\left[\left(\beta^S + \beta^G\right) - \left(\beta^S \varepsilon^S + \beta^G \varepsilon^G\right)\right]\right\} D_{t-1} - \left[1 - \dfrac{\delta'\left(\beta^S + \beta^G\right)}{(1+\lambda_1)Y^n}\right]\left(1 + r^D\right) D_{t-2} + \\ + \dfrac{\delta'}{(1+\lambda_1)Y^n}\left[\beta^S\left(A^S - Br^{min} - Y^n\right) - \beta^G\left(A^G - Br^{min} - Y^n\right)\right] + \dfrac{\delta'}{(1+\lambda_1)}\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right] \end{array} \tag{81}$$

Proposing a constant solution in (81) for D_t in the form D^*, the long-term equilibrium value of debt is obtained in this particular case[2]:

$$D^* = \frac{\beta^S\left(A^S - Br^{min} - Y^n\right) - \beta^G\left(A^G - Br^{min} - Y^n\right) + Y^n\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right]}{\left(\beta^S \varepsilon^S + \beta^G \varepsilon^G\right) - \left(\varepsilon^S + \varepsilon^G\right) r^D} \tag{82}$$

To analyse the stability of (82), we must determine the values of the characteristic roots b_1 and b_2 of the characteristic equation of the homogeneous equation from (81):

$$b_1, b_2 = \frac{\left\{2 + r^D - \frac{\delta'}{(1+\lambda_1)Y^n}\left[\left(\beta^S + \beta^G\right) - \left(\beta^S \varepsilon^S + \beta^G \varepsilon^G\right)\right]\right\}}{2} \pm \frac{\sqrt{\left\{2 + r^D - \frac{\delta'}{(1+\lambda_1)Y^n}\left[\left(\beta^S + \beta^G\right) - \left(\beta^S \varepsilon^S + \beta^G \varepsilon^G\right)\right]\right\}^2 - 4\left[1 - \frac{\delta'\left(\beta^S + \beta^G\right)}{(1+\lambda_1)Y^n}\right]\left(1 + r^D\right)}}{2} \quad (83)$$

Given the complexity of solving (83), we work with the inequalities proposed by Gandolfo (1976: 58); these are a set of necessary, sufficient conditions such that the roots b_1 and b_2, whether real or complex, are less than one in absolute value:

1st: $r^D < \frac{\left(\beta^S \varepsilon^S + \beta^G \varepsilon^G\right)}{\beta^S + \beta^G}$

2nd: $r^D < \frac{\delta'\left(\beta^S + \beta^G\right)}{(1+\lambda_1)Y^n - \delta'\left(\beta^S + \beta^G\right)}$ (84)

3rd: $r^D < \frac{4(1+\lambda_1)Y^n - 2\delta'\left(\beta^S + \beta^G\right) - \delta'\left(\beta^S \varepsilon^S + \beta^G \varepsilon^G\right)}{\delta'\left(\beta^S + \beta^G\right) - 2(1+\lambda_1)Y^n}$.

If the inequalities expressed in (84) are satisfied, the long-term value of foreign debt (82) is stable.

3.2.4 Explanation

When the union is formed, the central bank sets an interest rate that allows the inflation rate in the area to be 2%. Although adequate for the union as a whole, this interest rate is not adequate for its separate components (expression (69)). The inflation rate of each of the partners deviates from 2%, and it will be higher in the bloc with the higher natural interest rate or with more intense supply-side pressure (Spain). There will be differences in the competitive positions of the countries both from the internal and the external point of view, although the area's inflation rate and the exchange rate will remain unchanged.

The existence of inflationary pressures from the supply side opens the possibility that the aggregate output gap is not zero. In the absence of such tensions, the union will reach its potential income, and each bloc will separately show output gaps that are identical in quantity with opposite signs depending on their natural interest rates.

In the presence of supply tensions, the aggregate output gap will not be zero; instead, its magnitude will be given by the pressures of supply in each bloc, and the partners' output gaps will be determined both by its natural interest rate and by the value of pressures on the supply side.

Changes in the internal and external competitiveness of the blocs in the union lead to intra- and extra-union trade imbalances, with a negative sign for Spain and a positive sign for Germany; these, in turn, will reduce or increase, respectively, the income and inflation rates of each. Trade deficits that occur in each period in Spain are funded by continuous emission of a debt which, we assume, Germany buys outright.

In the absence of asymmetry, increased debt in Spain would reduce income and the inflation rate of this bloc to the same extent as these parameters would increase in Germany due to the enlargement of its asset portfolio. Inflation in the monetary area would not change, and the central bank would not change the interest rate (expression (69)). Inflation rates in the blocs would continue to vary in opposite directions until they coincide. At that moment, the open dynamic due to the mechanism of competitiveness would cease, and a steady state in which incomes will be those that guarantee inflation rates of 2%, the foreign debt will reached its equilibrium value (expression (61)) and the total trade surplus of Spain will enable it to cover the interests of the period of total debt accumulated (expression (73)) would be reached.

In the event of asymmetry, the growth of foreign debt would shrink the union's total demand, the area's inflation rate would be less than 2%, and the central bank would reduce the interest rate (expression (70)). This reduction in the interest rate would not completely neutralize the Spanish contraction. It would strengthen the German expansion, and the inflation rates of the blocs would end up being lower or higher than 2% until the competitiveness mechanism equalize them at 2%. The contracting effect of asymmetry would be compensated by reductions in the interest rate.

It is possible, for the interest rate run to end before the debtor country achieves a trade surplus that guarantees the stability of foreign debt. If this occurs, there would be a continuing decline in demand and income of the monetary union as debt increases because the contractionary drift would not be neutralized. If the equilibrium given by (61) is reached, inflation rates in the blocs would coincide (and be below 2%), competitiveness would no longer vary, Spain would obtain a trade surplus equal to the interests of the debt, and the incomes of the two countries would be below those guarantying the 2%. As a result of the differences between inflation rates in the union –now below 2%- and those in the rest of the world, an ongoing appreciation of the euro relative dollar would result. However, if the equilibrium value in (61) is not reached, inflation rates and the incomes of the two countries would be reduced without limit.

4 STATICS OF THE MODEL: A GRAPHICAL ANALYSIS

The purpose of this section is the graphical determination of the values of the endogenous variables in the steady state, i.e., when the following two equilibria are met: the inflation rates in the two countries are equal, and foreign debt has stabilized. In addition, a third equilibrium, i.e., that the central bank achieves its inflation target of 2%, may or may not be met whether the union falls or not into the trap. At the end of this section, we show that these equilibrium values are generally identical for the prices sub-model and for the general model. We can therefore use the results of the equilibrium of the prices sub-model to justify the proposals that will be stated in the following section and that are intended to refer to the general model. This naturally can be done provided that the equilibrium in the general model is reached.

The static analysis performed here is based on six equations. The first equation determines the combinations of the positive foreign balance of the debtor country and the value of the debt which ensure that the inflation rates of the two countries are equal. It is obtained by making $\pi^S = \pi^G$ in (12):

$$\frac{Y^n}{\beta}\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right] + Y^S - Y^G = 0 \qquad (85)$$

Replacing Y^S and Y^G in (85) with their values in (22) and (23), respectively, assuming $B^S = B^G = B$, and solving for the total positive foreign balance of Spain, X^{TS}, we can obtain the equation for the line $\pi^S = \pi^G$, whose slope is $\left(\varepsilon^S + \varepsilon^G\right)/2$:

$$X^{TS} = \frac{-\frac{Y^n}{\beta}\left[\left(\alpha^S - \dot{T}^S\right) - \left(\alpha^G - \dot{T}^G\right)\right] - \left(A^S - A^G\right) + \left(\varepsilon^S + \varepsilon^G\right)D}{2} \qquad (86)$$

The second equation determines the value of the total foreign trade balance of Spain that ensures that the foreign debt is stabilized. This value is obtained by making $D_t = D_{t-1} = D$ in (20), as follows:

$$X^{TS} = r^D D \tag{87}$$

The line that represents this equation, whose slope is r^D, is denoted $r^D D = X^{TS}$.

The third equation determines the values of the interest rate that ensures that the union's mean inflation is 2%. To obtain this equation, we make $\bar{\pi} = 0.02$ in (24); having assumed that $\pi^S = \pi^G$, the equation will be:

$$\pi^S = \pi^G = 0.02 \tag{88}$$

Later, (10) and (11) are added; the sum equals 0.04. Accepting that $\beta^S = \beta^G = \beta$ and $Y^{n,S} = Y^{n,G} = Y^n$, the value $\hat{Y}^S + \hat{Y}^G$ is obtained, i.e., the sum of the incomes that, for certain supply pressures, guarantee that the union's mean inflation is 2%. The central bank will achieve this inflation rate when $D = 0$ if it establishes an interest rate $\hat{r}^0$ such that

$$A^S + A^G - 2B\hat{r}^0 = \hat{Y}^S + \hat{Y}^G. \tag{89}$$

The bank will achieve this inflation rate for any value of debt D if the $\hat{r}$ that it establishes is

$$A^S + A^G - \left(\varepsilon^S - \varepsilon^G\right)D - 2B\hat{r} = \hat{Y}^S + \hat{Y}^G. \tag{90}$$

By equalizing (89) and (90), we obtain

$$\hat{r} = \hat{r}^0 - \frac{\left(\varepsilon^S - \varepsilon^G\right)}{2B} D. \tag{91}$$

The equations of equilibrium income of the countries (the fourth and fifth equations) are obtained by making $\pi^S = \pi^G$ in (10) and (11) and considering (88), as follows:

$$\hat{Y}^S = Y^n - \frac{Y^n}{\beta}\left(\alpha^S - \dot{T}^S\right) \tag{92}$$

$$\hat{Y}^G = Y^n - \frac{Y^n}{\beta}\left(\alpha^G - \dot{T}^G\right). \tag{93}$$

It is noted that the blocs' equilibrium incomes $\hat{Y}^S$ and $\hat{Y}^G$ are determined by the potential incomes and the supply-side pressures of the two countries.

The sixth and final equation shows the value of the foreign balance of the debtor country as the sum of its net exports to the creditor country and its exports to the rest of the world. This value is obtained by replacing $-X^{I}$ in (13) with the value obtained in (15) and replacing X^{S} with its value in (16), as follows:

$$X^{TS} = \delta_1 Ln\left[\left(\frac{Y^G}{Y^S}\right)^{j_1}\left(\frac{P^G}{P^S}\right)^{j_2}\right] + \delta_2 Ln\left[\left(v\frac{Y^W}{Y^S}\right)^{j_1}\left(\frac{P^W E}{P^S}\right)^{j_2}\right]. \quad (94)$$

Equations (86), (87), (91), (92), and (93) are represented graphically in panels (a), (b), and (c) of Figure 1. The lines $\pi^S = \pi^G$ and $X^{TS} = r^D D$ of panel (a) are equations (86) and (87), respectively. These lines intersect at point E and univocally determine the equilibrium values X^{TS*} and D^*. It is noted that if $\varepsilon^S = \varepsilon^G = 0$, the line $\pi^S = \pi^G$ becomes horizontal and achieving equilibrium is impossible unless the interest rate r^D is negative.

In panel (b), the interest rate that guarantees mean inflation in the union of 2% (equation (91)) is plotted. This interest rate does not depend on debt in the absence of asymmetry, $\varepsilon^S = \varepsilon^G$ (trajectory r_0), and decreases with debt if $\varepsilon^S > \varepsilon^G$ (trajectories r_1 and r_2). In either case, the interest rate is constant when $D = D^*$. If the interest rate reaches the minimum value $r_2 = r^{min}$, the economy of the monetary union will fall into the liquidity trap.

In panel (c), the equilibrium incomes are represented (expressions (92) and (93)). The values of these incomes depend on what happens in panel (b). While the central bank's interest rate can decrease to offset the contractionary effect of the debt, Y^S and Y^G will maintain their equilibrium values (values that guarantee inflation rates of 2% in each of the member countries). If the monetary union falls into the liquidity trap (as in panel (b), trajectory r_2), the asymmetry will cause a decrease in the union's total income; this decrease will be distributed among the partners to ensure equality in inflation rates. The income of the blocs will decrease until the foreign debt reaches equilibrium ($D = D^*$); at this point, the income values will stabilize again.

The general model differs from the prices sub-model only in that in expression (94) $j_1 = 0$ in tl X^{ts} ices sub-model and $j_1 \neq 0$ in the general model. The remaining equations are the same for both models; therefore, the equilibrium values are equal for all variables in both models. Due to the flexible nature of the variables

P^G/P^S, P^w/P^s and E, which will have different values in each of the models, the same value for X^{TS} can satisfy expression (94) in both models, with $j_1 = 0$ in the prices model and with $j_1 \neq 0$ in the general model. For this reason, to argue the proposals in the next section, we use the expressions of equilibrium obtained for the prices sub-model (expressions (61), (73), (74), and (75)) and expression (91) from the general model for $D = D^{*}$.

5 DISCUSSION: TEN PROPOSALS ON THE MONETARY UNION MODELLED

This section is intended to demonstrate the validity of the following ten proposals for the monetary union represented by what has been called the general model.

If we could ensure that the general model reaches equilibrium, if $\beta \neq 0$, and if the proposals relate to the behaviour of the union at steady state, the equilibrium results obtained in the execution of the prices sub-model could be used for justification, matching the results provided by the graphical analysis in the previous section.

The assurance that equilibrium is reached in the general model can only be obtained by simulation of each particular case, i.e., for specific combinations of parameters. Even accepting the lack of generality of the results thus obtained, a simulation exercise for each of the proposals is performed. This is done not only to show that in this case equilibrium is reached but also to further see that the display of trajectories of the variables contributes to the understanding of the illustrated proposal.

It is interesting to note that the initial values of the endogenous variables do not appear in the formulas that determine their equilibrium values, i.e., they do not affect them. This is important for the subsequent argument.

The results of the simulations are depicted in the figures accompanying the proposals, and the simulation experiments are summarized in Table 1. On the left side of this table, the specific values of the parameters that define each of the proposed scenarios of the general model are shown, and on the right side, the equilibrium values of the fundamental variables of the model obtained in each experiment are shown. The first row represents the initial equilibrium that will be used as a reference; the subsequent rows show how the values of the endogenous variables change with respect to their reference values.

Proposal 1. The problems addressed in this paper arise when, in the initial situation of the monetary union, the characteristics of the two member countries are such that

$$\left(\frac{Y^n}{B\beta}\right)\left(\alpha^S - \dot{T}^S\right) + r^{n,S} \neq \left(\frac{Y^n}{B\beta}\right)\left(\alpha^G - \dot{T}^G\right) + r^{n,G} \quad (95)$$

,

i.e., when the monetary union is not what we would call *an ideal monetary union*. When the two members in (95) are equal, there are no differences in the inflationary pressures within the blocs, the interest rate that the central bank sets and the foreign exchange rate are suitable for both member countries, and there are no differences in competitiveness between the members. In this case, the steady state is achieved from the outset.

Expression (95) is obtained by making $\pi_0^S = \pi_0^G$ in (12); considering that $D_0 = 0$, $X_0^{TS} = 0$, and $B^S = B^G = B$, we have $Y_0^S - Y_0^G = A^S - A^G$. Finally, definitions of the natural interest rate are used (equations 26) and (27)).

Row 1 of Table 1 shows, on the left side, a combination of parameters of the two countries that ensures compliance with (95), and, on the right side, that inflation rates are equal and foreign balances and debt are zero, i.e., a case of an ideal monetary union.

Proposal 2. The member country that corresponds to the higher side of inequality

$$\left(\frac{Y^n}{B\beta}\right)\left(\alpha^S - \dot{T}^S\right) + r^{n,S} > \left(\frac{Y^n}{B\beta}\right)\left(\alpha^G - \dot{T}^G\right) + r^{n,G} \quad (96)$$

will be the debtor country, and vice versa.

The bloc for which equation (96) is greater will be the debtor. This bloc will have a higher inflation rate, its competitive position will deteriorate, and net trade flows with the outside will be negative, resulting in the appearance of foreign debt stocks.

This statement is illustrated by simulating a scenario in which, when the monetary union is formed, the countries have differences in natural interest rates, identical supply conditions, and $\varepsilon^S = \varepsilon^G = 0.3$ (Table 1, Row 2). In Figure 2 (a), it is noted that the bloc with the higher inflation rate at the beginning (Spain) loses competitiveness, acquires deficits, and goes into debt (Figure 2 (b)).

Proposal 3. Provided that the interest rate on the foreign debt is positive ($r^D > 0$), there will be what is called 'inversion' of the initial situation: the country that initially began suffering a current account deficit will eventually in the final equilibrium achieve a surplus, and vice versa.

The debtor country is the country that satisfies (96). At the steady state, its foreign debt will be positive (expression (61)), and the interests of the country must be covered entirely by a trade surplus (expression (73)) to make stabilization of the stock possible.

The simulation experiment described in proposal 2 is used to illustrate proposal 3 (Table 1, Row 2). In Figure 2 (b), we see that at steady state the total trade balance of the initial debtor country is eventually reversed.

Proposal 4. If $\varepsilon^S = \varepsilon^G = 0$ and $r^D \geq 0$, the monetary union will have an explosive character. If $\varepsilon^S = \varepsilon^G = 0$ and $r^D < 0$, the monetary union will reach a steady state with a deficit for the debtor country and a surplus for the creditor country.

In the first case, if in (61) we make $\varepsilon^S = \varepsilon^G = 0$ and $r^D \geq 0$, the long-term value of debt would tend to infinity ($D \rightarrow \infty$). In the second case, when $\varepsilon^S = \varepsilon^G = 0$ and $r^D < 0$ in (61), $D^* > 0$, and the initial debtor country will maintain the foreign deficit in the long term (expression (73)). The equilibrium incomes will be given by (74) and (75), and the inflation rates will be 2%.

The experiment that is performed to show this proposal is one in which, in the initial situation, debt and assets have no effect on the aggregate demand of the blocs ($\varepsilon^S = \varepsilon^G = 0$) and in which the partners differ in their demand and supply conditions. Given this context, two possible scenarios arise depending on whether the interest rate on the debt is positive ($r^D \geq 0$) or negative ($r^D < 0$). In the first case, $r^D \geq 0$ (Table 1, Row 3.1), we reach a situation in which the inflation rates are equal at 2%. The equilibrium income needed to achieve this result is achieved with the foreign deficits that appear on the right side of Table 1, and foreign debt grows without limit (Figure 3 (a)). When the debt accrues negative interest ($r^D < 0$, Table 1, Row 3.2), this stock is stabilized over the long term for a negative value of the trade balance for the debtor country (Figure 3 (b)).

Proposal 5. For the economy to fall into the liquidity trap, it is a necessary condition that $\varepsilon^S > \varepsilon^G$, i.e., that there is asymmetry, and it is a sufficient condition that the equilibrium interest rate is lower than the minimum interest rate, i.e., that $\hat{r}^* < r^{\min}$.

To justify this proposal, we make $D = D^*$ in (91):

$$\hat{r}^* = \hat{r}^0 - \frac{(\varepsilon^S - \varepsilon^G)}{2B} D^* \tag{97}$$

If in (97) $\varepsilon^S = \varepsilon^G$ and $\hat{r}^0 \geq r^{\min}$, $\hat{r}^*$ will be greater than $r^{\min}$, and the union's economy will not fall into the liquidity trap. If there is asymmetry $\varepsilon^S > \varepsilon^G$ such that the economy falls into the trap, it will be a sufficient condition that $\hat{r}^* < r^{min}$, i.e., $\hat{r}^* = \hat{r}^0 - \frac{(\varepsilon^S - \varepsilon^G)}{2B} D^* < r^{\min}$. The interest rate will have reached its minimum value before the debt stops growing, and when neutralization of its contractive effect ceases, inflation rates will be below 2%, and incomes will have values lower than those that otherwise would have achieved these rates of inflation.

To illustrate this proposal, three situations are considered that describe the start of the monetary union: one in which the necessary condition is not met (Table 1, Row 4.1), one in which the necessary condition is met but is not sufficient (Table 1, Row 4.2), and a final situation in which both conditions are met (Table 1, Row 4.3). Figure 4 (a) shows that when the necessary condition $\varepsilon^S = \varepsilon^G$ is not met, the union's economy does not fall into the liquidity trap. Here, we see that as time passes and the debt grows, the interest rate established by the central bank remains at its initial value ($\hat{r}_t = \hat{r}_0$).Figure 4 (b) shows that although the necessary condition $\varepsilon^S > \varepsilon^G$ is met, there is a contractionary drift, and the interest rate of the central bank decreases ($\hat{r}_t < \hat{r}_0$),debt reaches its equilibrium value D^* before the interest rate is made less than the minimum ($\hat{r}^* > r^{min}$); that is, the sufficient condition is not met. Finally, Figure 4 (c) shows the path of the interest rate that would ensure inflation of 2% ($\hat{r}^*$), a path that will be truncated by the restriction of $r^{\min}$.

Proposal 6. The total equilibrium deficits/surpluses of the two member countries do not depend on their equilibrium incomes, their price elasticities, or their income elasticities.

The surplus of the debtor country should be equal to the interest on the equilibrium debt so that the debt stops growing; in the determination of this debt (expression (61)), income, price elasticity, and income elasticity have no influence.

Where D^* is equilibrium debt, the value of the positive balance of the debtor country will be given by (73), and equality (94) will be achieved for each set of values j_1, j_2, Y^G, Y^S through the necessary changes of $\frac{P^G}{P^S}$, $\frac{P^W}{P^S}$ and E, which will act as accommodative variables.

To illustrate this proposal, we assume a scenario in which the values of the initial characteristic parameters of the blocs are consistent with those of a previous experiment (corresponding to Row 4.1) except for j_1 and j_2 (Row 5 of Table 1). One can see that in both cases a steady state is reached in which the values of the endogenous variables are identical, whereas the changes in the exchange rate in the two experiments differ (Figure 5 (a) and (b)).

Proposal 7. It is sufficient that one of the member countries meet the conditions we propose to define it as a parasite country and that $r^D > 0$; ***under these conditions, whatever the characteristics of the other member country, the union falls into the liquidity trap.***

In this model, the creditor country will be called 'the parasite country' when it satisfies the following conditions. First, asymmetry is absolute, meaning $\varepsilon^G = 0$; second, that, at the time when the monetary union is established for the minimum interest rate, a positive total foreign balance $\hat{X}_0^{TG}$ exists such that its equilibrium income guarantees an inflation rate of 2%. That is, it satisfies

$$A^G - Br^{\min} + \hat{X}_0^{TG} = Y^{n,G}\left[1 - \frac{\left(\alpha^G - \dot{T}^G\right)}{\beta}\right]. \qquad (98)$$

The name 'parasite country' can be justified by the extreme dependence of the creditor bloc on foreign demand to achieve an income compatible with 2% inflation.

The rationale for this proposal is as follows: the interest rate that would ensure that in the steady state the union does not fall into the liquidity trap is $\hat{r}^*$ in the following expression:

$$A^G + \varepsilon^G D^* - B\hat{r}^* - r^D D^* = Y^{n,G}\left[1 - \frac{\left(\alpha^G - \dot{T}^G\right)}{\beta}\right]. \tag{99}$$

But, to make $\hat{r}^*$ an achievable interest rate, we must have $\hat{r}^* \geq r^{min}$. Now, subtracting expression (98) from (99), we obtain

$$\hat{r}^* = r^{min} - \frac{\hat{X}_0^{TG} - \left(\varepsilon^G - r^D\right)D^*}{B}. \tag{100}$$

Because we have assumed that $\hat{X}_0^{TG}$ and r^D are positive, D^*, denoting the foreign assets of the creditor country, will also be positive, and, according to the definition of 'parasite country', $\varepsilon^G = 0$. We check the equilibrium interest rate is lower than the minimum ($\hat{r}^* < r^{min}$), then it is inevitable that the monetary union falls into the liquidity trap.

To illustrate this proposal, we show through simulation that if the conditions required by the theorem are met, however favourable the characteristics of the debtor country, the union will still fall into the trap (Table 1, Row 6). The first condition of the theorem will be fulfilled by making $\varepsilon^G = 0$. The second condition, that the creditor country needs a surplus to achieve potential income with the minimum interest rate, is equivalent to the situation in which the natural rate of interest is below the minimum ($r^{n,G} < r^{\min}$). Therefore, as Germany's natural rate of interest, we introduce a rate of -0.03, less than -0.02, which is the minimum. Fulfilling these two conditions, although the debtor country's demand is very strong ($r^{n,S} = 0.13$), its supply tensions are also strong ($\left(\alpha^S - \dot{T}^S\right) = 0.02$), and the interest rate of the debt is very low ($r^D = 0.02$), the right side of Row 6 of Table 1 shows that the union will fall into the trap. Figure 6, panels (a) and (b), also illustrates that the monetary union will fall into the liquidity trap regardless of the conditions specified above for the debtor country.

Proposal 8. If we accept the hypothesis that quantity reactions occur more rapidly than price reactions, in the short term, the situation that the monetary union reaches can be described by the equilibrium that is achieved in the quantities sub-model, assuming that this situation is hardly changed by the slowness of the price mechanism. However, over a sufficiently long period,

whatever the result of the performance of the quantities sub-model, the final equilibrium will be the same as would have been reached by the exclusive performance of the prices sub-model. In this sense, there would have been a predominance in the short term of quantity reactions, and there would be a predominance of the price mechanism in the long term.

The justification for this proposal is that the equilibrium that is achieved in the quantities sub-model, which, when the interest rate on the debt is positive, requires that the income of the debtor country be lower than that of the creditor country, is not any more an equilibrium when the price mechanism is activated. Because at that moment different inflation rates will appear in the two member countries, and their relative competitiveness will vary.

Moreover, income and the other variables resulting from the quantities equilibrium can be considered as the initial values of variables when the prices model begins; and as already argued, the final equilibrium do not depend of these initial values of variables.

A simulation exercise is conducted in which we assume a monetary union that consists of two blocs whose natural interest rates differ and who have identical conditions of supply and do not present asymmetry in the initial situation. The quantity reactions operate throughout the entire time path from $t=0$ until $t=70$ (Table 1, Row 7.1), whereas price reactions begin acting after $t=35$ (Table 1, Row 7.2). It is noted that the equilibrium values that are obtained from the exclusive operation of the quantities mechanism (Table 1, Row 7.1) are no longer the equilibrium values when the price mechanism is activated at $t=35$ and that the equilibrium given by the price reactions ultimately prevails (Table 1, Row 7.2). In Figure 7, panel (a), we see that the exclusive operation of the quantities mechanism leads to equilibrium values for the blocs' incomes that do not coincide with their potential incomes**,** despite the lack of differences in the countries' supply conditions**,** and that such gaps end as a result of the price reactions. At the same time, differences in inflation rates that would result from the sole intervention of the quantities mechanism (period $t=34$) disappear if the use of price reactions is allowed (Figure 7, panel (b)).

Proposal 9. If at any point of the trajectory of the monetary union (or even when it has reached the steady state) the values of the fundamental parameters A^S***,*** A^G***,*** $\left(\alpha^S - \dot{T}^S\right)$***,*** $\left(\alpha^G - \dot{T}^G\right)$***,*** ε^S***,*** ε^G ***and*** r^D ***are changed, the new***

equilibrium that is reached does not depend on the path followed prior to that point or on the initial parameter values. It depends solely on the new values of these parameters.

To justify this proposal, it is important to remember that in the general model the initial values do not influence the final equilibrium and to consider the values of the endogenous variables when changing parameters such as the initial conditions.

This property allows us to argue that an economic policy designed to achieve the ideal monetary union is the same at any stage of the union and that it consists simply of converting the values of the parameters to those that initially would have resulted in an ideal monetary union, i.e., the values that ensure compliance with (95), an equation that we can transform by making the natural interest rates $r^{n,S}$ and $r^{n,G}$ in A^S and A^G. This would be

$$\left(\frac{Y^n}{\beta}\right)\left(\alpha^S - \dot{T}^S\right) + A^S = \left(\frac{Y^n}{\beta}\right)\left(\alpha^G - \dot{T}^G\right) + A^G \qquad (101)$$

The economic policy we propose will therefore be that which, through action agreed upon by the two countries, changes A^S, A^G, α^S, and α^G such that, at all times, equation (101) is satisfied. The debtor country should implement policies that reduce A^S (contractionary fiscal policies) and/or policies of wage restraint, and the creditor country should implement the contrary policies. It seems reasonable to propose that the adjustment is made to a greater extent by the expansive and wage stimulus policies implemented by the creditor country because these will be more politically feasible and will generate fewer side effects. Additionally, we could define a mercantilist strategy as contrary to the latter suggestion: the creditor country reduces wage growth and shrinks its domestic aggregate demand to achieve a foreign surplus that guarantees sufficient aggregate demand and will force the other bloc to implement contractionary fiscal policies and promote wage moderation (internal devaluation) as a means of controlling debt growth.

In the simulation of the general model, we considered a monetary union in which, at the time of its constitution, the blocs have discrepancies in their demand conditions[3] that cause interaction between flows and stocks and therefore result in the accumulation of foreign debt (Table 1, Row 8.1). It is assumed that the economic authorities of the blocs intervene in a coordinated manner and, in period 25, adjust the magnitude of the characteristic parameters to those that, had they been present originally, would have allowed the steady state, which has been called ideal (Table 1

Row 8.2). Such intervention would allow the monetary union to achieve a long-term equilibrium characterized by a null value of foreign debt (Figure 8, panels (a), (b), and (c)).

Proposal 10. Modifying the amount of the debt or eliminating it through restructuring, payment, or remission without changing the basic parameters of the monetary union is an ephemeral solution because after the decision is made the same dynamic trajectory that brought the debt to the value that is being reduced or eliminated will be repeated.

The justification for this proposal is simple. It should be understood that forgiveness, restructuring, or payment consists of changing the value of debt and other related endogenous variables at the time of the intervention, keeping the final destination of the union written in the values of the fundamental parameters.

An exception to this proposal is that in some cases debt restructuring takes the form of reductions in interest rates, which is a fundamental parameter in this model. If restructuring adopts this form and the interest rate reduction is permanent, the dynamics of the monetary union would change.

The scenario simulated to illustrate this proposal is one in which the blocs of the monetary union initially have discrepancies in their demand conditions (Table 1, Row 9); these initial characteristics are identical to those in the experiment described in the prior proposal (Table 1, Row 8.1). If, once this equilibrium is reached, it is assumed that the debt is forgiven in full (in $t = 35$)), the process that began when the union was formed would be replicated, and the steady state that remission disturbed would be achieved again (Figure 9 panels (a), (b), and (c)).

6 CONCLUSIONS

The ideal monetary union is one in which no surpluses or deficits between the member countries and external countries exist either at the formation of the union or later on. This result is achieved when the natural interest rates and supply pressures of both countries taken together ensure that inflationary pressure is the same in both. That one of the two countries adopts neo-mercantilist strategies (reducing α with respect to $\dot{T}$) is sufficient to make this ideal condition not possible. However, this strategies are not necessary, it is sufficient that demand conditions differ for the two countries to also make this ideal monetary union impossible.

The problems that arise when this initial equality of inflationary pressures is not met will be amplified by the actions of the following four mechanisms:

1) Interest rate on debt. When the union reaches a steady state, the creditor country should accept a trade deficit equal to the interest on its accumulated debt. The higher the interest rate on foreign debt, the greater the equilibrium deficit will be and the more difficult the adjustment for the creditor country that is becoming the deficit country to accept what we have called the inversion of the initial situation.
2) Degree of asymmetry. The higher the degree of asymmetry ($\left(\varepsilon^{S}-\varepsilon^{G}\right)$), the greater the contractionary result that the monetary union would suffer and the greater the probability that the central bank's interest rate is made zero before the debt stabilizes. A high interest rate, which makes the debt grow faster, would also accentuate this danger. It seems reasonable to say that a monetary union doomed to fall into the liquidity trap is an unviable union.
3) Predominance of quantity reactions. The equilibrium that characterizes the steady state is reached by reducing the deficits of the debtor country. This reduction can be achieved by implementing quantity reactions that involve the contraction of the economy of the debtor country and its imports, or, where price reactions predominate, that improve price competitiveness. The magnitude and duration of the adjustment (contraction of income and employment) will be greater when quantity reactions predominate over price reactions.
4) Impact of debt on aggregate demands of member countries. For the union to reach a steady state, it is necessary that the foreign debt/assets of the two member countries stop growing; to this end, the demand, income, inflation rates, and imports of the debtor/creditor countries must decrease/grow. The parameters that determine the effects of debt on these variables are ε^{G} and ε^{S}. The greater $\varepsilon^{G}+\varepsilon^{S}$, the lower the required debt will be for the union to reach its final equilibrium. *Sensu contrario*, if $\varepsilon^{G}+\varepsilon^{S}$ is lower than the interest rate on the debt, this debt would be explosive.

The operation of the union resulting from the endogenous mechanisms explained can be corrected by actions of economic policy. Economic policies should take into account the following three considerations:

a) The austerity policy applied by the debtor country can be understood in this context as a reduction in demand induced by public debt that, in part, will become foreign debt and will be achieved through public deficit control. The problem with this policy is that if the burden of the adjustment falls mainly on the debtor country (if ε^G is very low) and if quantity reactions predominate, the contraction the debtor country suffers will be deep and lasting.

b) If the fundamentals parameters of the monetary union are not changed, payment or forgiveness of the debt will be a short-lived solution because it will lead to a repetition of the process that spawned the debt.

c) Changing the parameters is equivalent to reducing the differences between the supply tensions or natural interest rates of the two countries, and this reduction should be achieved through demand policies and wage policies that should be established through cooperative agreement.

Funding: This work was supported by Departamento de Dirección de Empresas e Historia Económica (Universidad de La Laguna).

Acknowledgements: We would like to express our thanks to Félix Marcos for his exhaustive reading of this article, the rigor of his criticism, and the encouragement that his confidence in our work has given us. We must also thank Luis Corchón, whose suggestions significantly changed the orientation of this paper.

REFERENCES

Bibow, J. “Making the euro viable: the Euro Treasury Plan.” Levy Economics Institute Working Papers Series Paper, 2015, 842.

Caiani, A.; Godin, A., and Lucarelli, S. “A Stock Flow Consistent Analysis of a Schumpeterian Innovation Economy.” *Metroeconomica*, 2014a, 65, 397–429.

Caiani, A.; Godin, A., and Lucarelli, S. “Innovation and Finance: a stock flow consistent analysis of great surges of development.” *Journal of Evolutionary Economics*, 2014b, 24, 421–448.

Caverzasi, E. and Godin, A. “Post-Keynesian stock-flow consistent modelling: a survey.” *Cambridge Journal of Economics*, 2015, 39, 157–187.

Comunale, M. and Hessel, J. “Current account imbalances in the Euro area: competitiveness or financial cycle?” DNB Working Paper, 2014, 443.

De Grauwe, P. *Economics of Monetary Union*. Oxford: Oxford University Press, 2016.

Dos Santos, C.H. “Keynesian theorising during hard times: stock-flow consistent models as an unexplored 'frontier' of Keynesian macroeconomics.” *Cambridge Journal of Economics,* 2006, 30, 541–565.

Flassbeck, H. and Spiecker, F. “The euro — a story of misunderstanding.” *Intereconomics*, 2011, 46, 180–187.

Fleming, J.M. “On Exchange Rate Unification.” *The Economic Journal*, 1971, 81, 467–488.

Gandolfo, G. *Métodos y modelos matemáticos de la dinámica económica*. Madrid: Editorial Tecnos, 1976

Gaulier, G. and Vicard, V. “Current account imbalances in the Euro area: competitiveness or demand shock.” *Banque de France Quarterly Selection of Articles*, 2012, 27, 5–26.

Godley, W. and Lavoie, M. “Two-country stock-flow-consistent macroeconomics using a closed model within a dollar exchange regime.” University of Cambridge Working Paper, 2003, 10.

Godley, W. and Lavoie, M. “Simple open economy macro with comprehensive accounting: a two country model.” University of Cambridge Working Paper, 2005, 20.

Godley, W. and Lavoie, M. *Monetary Economics: An Integrated Approach to Credit, Money, Income, Production and Wealth*. Hampshire: Palgrave Macmillan, 2007.

Kenen, P.A. “Theory of optimum currency areas: an eclectic view”. In R.A Mundell and A. Swoboda (eds.), *Monetary Problems of the International Economy*. Chicago: The University of Chicago Press, 1969, pp. 41–60.

Koo, R.C. *The Holy Grail of Macroeconomics: Lessons from Japan's Great Recession*. Singapore: John Wiley & Sons, 2008.

Magnifico, G.. *European Monetary Unification*. United Kingdom: Palgrave Macmillan, 1973.

McKinnon, R.I.. “Optimum currency areas.” *American Economic Review*, 1963, 53, 717–725.

Mundell, R. A. “Theory of optimum currency areas.” *American Economic Review*. 1961, 51, 657–665.

Storm, S. and Naastepad, C. “Myths, Mix-ups and Mishandlings: What Caused the Eurozone Crisis?” Annual Conference Institute for New Economic Thinking “Liberté, Égalité, Fragilité”, 2015, Paris, 11 April,

Taylor, L. “A foxy hedgehog: Wynne Godley and macroeconomic modelling.” *Cambridge Journal of Economics*, 2008, 32 (4), 639–663.

Universidad de La Laguna
Oficina de Sede Electrónica
Entrada
Nº registro: 2017/54291
Nº reg. oficina: OF002/2017/43067
Fecha: 11/11/2017 09:41:40